Today: Questions on 2.6 and do 2.7

$\tan(x-y) = \frac{y}{x^2+5} = \frac{f}{g}$ §2.6 #5

$$(\sec^2(x-y))(1-y') = \frac{y'(x^2+5) - y(2x)}{(x^2+5)^2} = \frac{f'g-fg'}{g^2}$$

$$-y'\sec^2(x-y) + \sec^2(x-y) = y'\left(\frac{x^2+5}{(x^2+5)^2}\right) - \frac{2xy}{(x^2+5)^2}$$

$$-y'\sec^2(x-y) - y'\left(\frac{1}{x^2+5}\right) = -\sec^2(x-y) - \frac{2xy}{(x^2+5)^2}$$

$$y'\left(\sec^2(x-y) + \frac{1}{x^2+5}\right) = \sec^2(x-y) + \frac{2xy}{(x^2+5)^2}$$

$$y' = \frac{\frac{\sec^2(x-y)}{1} \cdot \frac{(x^2+5)^2}{(x^2+5)^2} + \frac{2xy}{(x^2+5)^2}}{\frac{(x^2+5)\sec^2(x-y) + 1}{x^2+5}}$$

$$= \frac{(x^2+5)^2\sec^2(x-y) + 2xy}{(x^2+5)^{\cancel{2}}} \cdot \frac{\cancel{x^2+5}}{(x^2+5)\sec^2(x-y)+1}$$

$$= \frac{(x^2+5)^2\sec^2(x-y) + 2xy}{(x^2+5)^2\sec^2(x-y) + (x^2+5)}$$

My version:

$$\frac{(x^2+3)^2\sec^2(x-y) + 2xy}{(x^2+3)^2\sec^2(x-y) + (x^2+3)}$$

Same #5 using Product Rule

$$\tan(x-y) = \frac{y}{x^2+5} \implies$$

$$(fg)' = f'g + fg'$$

$$(x^2+5)\tan(x-y) = y \implies \frac{d}{dx}[y]$$

$$2x\tan(x-y) + (x^2+5)\sec^2(x-y)(1-y')$$

$$= 2x\tan(x-y) + (x^2+5)\sec^2(x-y) - (x^2+5)\sec^2(x-y)y' = y'$$

$$\implies y' + (x^2+5)\sec^2(x-y)\,y' = 2x\tan(x-y) + (x^2+5)\sec^2(x-y)$$

$$\implies y' = \frac{2x\tan(x-y) + (x^2+5)\sec^2(x-y)}{1 + (x^2+5)\sec^2(x-y)}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \qquad (fg)' = f'g + fg'$$

$$\frac{gf' - fg'}{g^2} = \frac{g\frac{df}{dx} - f\frac{dg}{dx}}{g^2} = \frac{\text{Low d-high} - \text{high d low}}{\text{low}^2}$$

2.6 Due Midnight
2.7 " " tomorrow 10/5
2.8 " " ~~Th~~ Wed 8/6
2.9 " " Fri

§2.7 we left off with Newton:

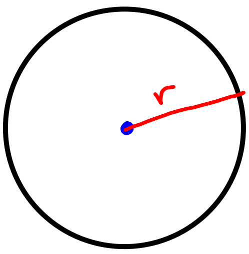
$$s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$$

$$s'(t) = -gt + v_0 = v(t)$$

$$s''(t) = -g = a(t) = v'(t)$$

4. 0/3 points SCalc8 2.7.014. [3354421]

A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 40 cm/s. Find the rate at which the area within the circle is increasing after each of the following.



given $\frac{dr}{dt} = 40\ \frac{cm}{s}$ $\quad t = 1, 5, 6$ secs

want $\frac{dA}{dt}$

$r = r(t) = 40\ \frac{cm}{s}\ t$

$r(1) = 40$

$r(5) = 200$

$r(6) = 240$

$A = \pi r^2$

$$A'(r) = \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$A'(40) = 2\pi(40)(40) = 3200\pi\ \frac{cm^2}{s}$$

$$A'(200) = 2\pi(200)(40) = 16000\pi\ \frac{cm^2}{s}$$

$$A'(240) = 2\pi(240)(40) =$$

$A(r) = A(r(t)) = \pi r^2(t) = \pi (40t)^2 = 1600\pi t^2$

Get all the way down to t!

~~$A(t) = 1600\pi t^2$~~

~~$A(1) = 1600\pi \frac{cm^2}{s}$~~

~~$A(5) = 1600(\pi)(25) =$~~

~~These are areas A~~

~~want $\frac{dA}{dt}$ = rate of change of area.~~

$A(t) = 1600\pi t^2 \Rightarrow$

$A'(t) = 3200\pi t$

$A'(1) = 3200\pi \frac{cm^2}{s}$

$A'(5) = 3200\pi(5) = 16000\pi \frac{cm^2}{s}$

$A'(6) = 3200\pi(6) = 192{,}000\pi \frac{cm^2}{s}$

$\rightarrow 19{,}200\pi \frac{cm^2}{s}$

$$\begin{array}{r} 32 \\ \times 6 \\ \hline 192000 \end{array}$$

I'm off by an order of magnitude.

§2.7

Ex) Let $C = C(t)$ = Caribou population numbers,
$W = W(t)$ = # of wolves, and
t = time in years

$$\frac{dC}{dt} = aC - bCW \qquad \frac{dW}{dt} = -cW + dCW.$$

$a = .09,\ b = .001,\ c = .09,\ d = .0001$

We find Pops that lead to stable pop.

$$\frac{dC}{dt} = .09C - .001CW \overset{\text{SET}}{=} 0$$

Nonlinear System

$$\frac{dW}{dt} = -.09W + .0001CW \overset{\text{SET}}{=} 0$$

$.09C = .001CW$

$.09 = .001W$

$90 = \frac{.09}{.001} = W$

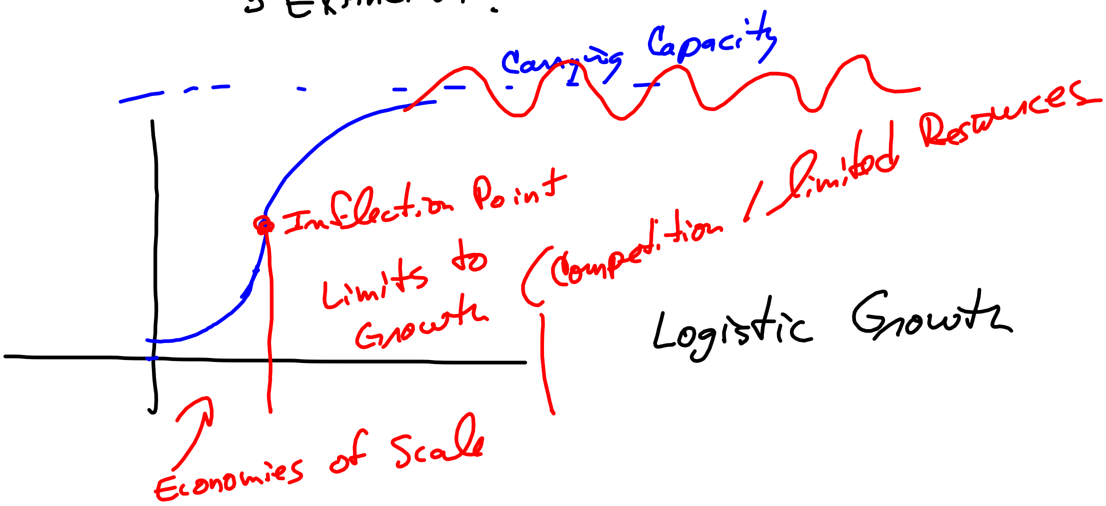
$-.09W + .0001WC = 0$

$.09W = .0001CW$

$\frac{.09}{.0001} = C = 900$

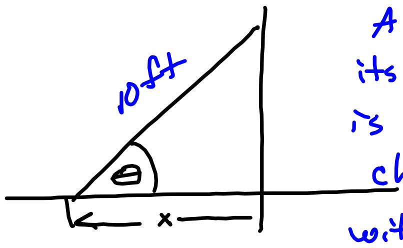
$(C, W) = (900, 90)$

OR $(C, W) = (0, 0)$ Extinction!



0/1 points SCalc8 2.8.032. [3426404]

A ladder 10 ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 0.8 ft/s, how fast is the angle between the ladder and the ground changing when the bottom of the ladder is 6 ft from the wall? (That is, find the angle's rate of change when the bottom of the ladder is 6 ft from the wall.)



$\frac{dx}{dt} = .8 \frac{ft}{s}$

A 10-ft ladder is sliding. The rate its base is sliding away from the wall is .8 ft/s. We want the rate of change the angle the ladder makes with the ground when the base is 6 ft from the wall

Let x = distance of the base from the wall, in ft;

θ = angle the ladder makes with the ground (in radians)

t = time, in seconds

We Want $\left.\frac{d\theta}{dt}\right|_{x=6}$

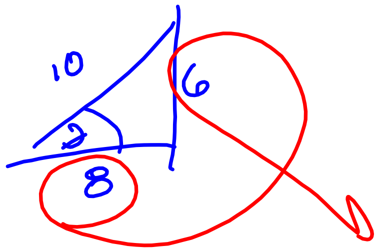
By Picture, $\frac{x}{10} = \cos\theta$

$\Longrightarrow$ $x = 10\cos\theta$. Differentiate wrt "t"

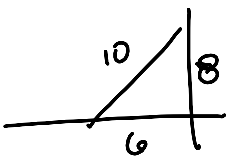
$\frac{dx}{dt} = (-10\sin\theta)\frac{d\theta}{dt} \Longrightarrow$

$$\left.\frac{d\theta}{dt}\right|_{x=8} = \left.\frac{\frac{dx}{dt}}{-10\sin\theta}\right|_{x=8} = \left.\frac{.8}{-10\sin\theta}\right|_{x=8} = \frac{.8}{-10\cdot\frac{3}{5}} = \frac{.8}{-6} = \frac{-8}{60} = \frac{-4}{30} = -\frac{2}{15}\ \frac{rad}{s}$$

$$\boxed{\left.\frac{d\theta}{dt}\right|_{x=8} = -.13\ \frac{rad}{s}}$$



No. x=6!



$$\frac{0.8}{-10\sin\theta} = \frac{0.8}{-10\cdot\frac{8}{10}} = \frac{0.8}{8} = .1 = \frac{1}{10}$$

So $\boxed{\left.\frac{d\theta}{dt}\right|_{x=6} = .1\ \frac{radians}{s}}$