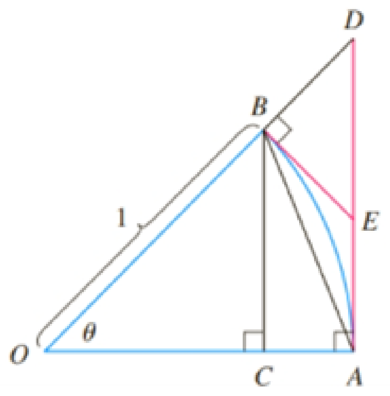



$$\frac{|BC|}{r} = \sin\theta < \text{arc } AB = \theta \Longrightarrow$$

$$\frac{\sin\theta}{\theta} < 1$$

$$\tan\theta > \theta$$

$$\theta < \frac{\sin\theta}{\cos\theta} \Longrightarrow$$

$$\cos\theta < \frac{\sin\theta}{\theta}$$

$$\frac{\sin(x+h) - \sin(x)}{h}$$

$$\sin(u+v) = \sin(u)\cos(v) + \sin(v)\cos(u)$$
$$\cos(u+v) = \cos(u)\cos(v) - \sin(u)\sin(v)$$

$$= \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h} = \frac{\sin(x)(\cos(h)-1) + \sin(h)\cos(x)}{h}$$

$$\left(\frac{\cos(h)-1}{h}\right)(\sin(x)) + \frac{\sin(h)}{h}\cos(x) \xrightarrow{h\to 0} \cos(x)$$

1 as $h \to 0$ b/c

$$\cos(h) < \frac{\sin(h)}{h} < 1$$

$$h \to 0$$

$$1 \le \lim_{h\to 0} \frac{\sin(h)}{h} \le 1$$

$$\left(\frac{\cos(h)-1}{h}\right)\left(\frac{\cos(h)+1}{\cos(h)+1}\right)$$

$$= \frac{\cos^2(h) - 1}{h(\cos(h)+1)} = \frac{-\sin^2(h)}{h(\cos(h)+1)}$$

$$= -\frac{\sin(h)}{h} \cdot \underbrace{\frac{\sin(h)}{\cos(h)+1}}_{1+1} \xrightarrow{h\to 0} -1 \cdot \frac{0}{2}$$

$$= 0 !$$

1 as $h \to 0$

$$1 - \cos^2(x) = \sin^2(x)$$
$$\cos^2(x) - 1 = -\sin^2(x)$$

Test 2 Part 1: Wednesday, 9/29. Covers 2.1 - 2.5.

$\frac{d}{dx}[\sin(x)] = \cos(x)$ $\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x)$ $\frac{d}{dx}[\cot(x)] = -\csc^2(x)$

$\frac{d}{dx}[\sec(x)] = \sec(x)\tan(x)$ $\frac{d}{dx}[\csc(x)] = -\csc(x)\cot(x)$

$$\frac{d}{dx}[\overset{f}{\sin(x)}\,\overset{g}{\cos(x)}] = \overset{f'g}{\cos(x)\cos(x)} \overset{+}{+} \overset{fg'}{\sin(x)(-\sin(x))}$$

$$= \boxed{\cos^2(x) - \sin^2(x)} = \text{All kinds of things.}$$

is fine

$$= 1 - 2\sin^2(x) = 2\cos^2(x) - 1 = \cos(2x)$$

TRIG IDENTITIES (From Test Video(s) for Trig, in the homework videos for each chapter):

https://harryzaims.com/122/videos/chapter-02/test-2/cheat-sheet-test-2.pdf

## Next Level

$(fg)' = f'g + fg'$

$$\frac{d}{dx}\left[(x^2+3x)\tan(x)\right] = (2x+3)\tan(x) + (x^2+3x)\sec^2(x)$$

## NEXT Level Chain Rule!

$\boxed{\S 2.5}$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$$\frac{d}{dx}\left[\sec(x^3)\right] = \underline{\sec(x^3)\tan(x^3)}\cdot 3x^2$$

Differentiating wrt $x^3$

$u = x^3$

$$\frac{d}{dx}\left[\sec(u)\right] = \sec(u)\tan(u)\cdot\frac{du}{dx}$$

$$= \sec(x^3)\tan(x^3)\cdot 3x^2$$

---

Didn't I just say that

$\frac{d}{dx}\left[\tan(\theta)\right] = 0$ ? This is correct, If you assume $\theta$ is independent of the variable $x$.

Now, if $\theta$ is $\theta(x)$, i.e., a function of $x$, then

$$\frac{d}{dx}\left[\tan(\theta(x))\right] = \sec(\theta)\tan(\theta)\cdot\theta'(x)$$

Typically, we DON'T assume $\theta$ is a function of $x$, unless somebody says to assume it is.

Trig Identities Presentation by Victoria Silva:

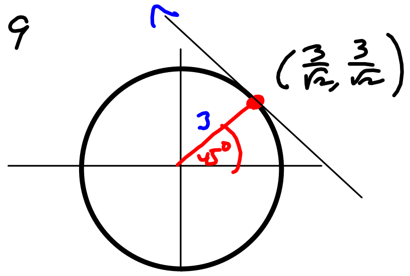
https://docs.google.com/presentation/d/1r12JSpa0fDnlO2wCEwBxaO4y2sOTv1KhPwstcoxBDtl/edit#slide=id.ge8825d009f_0_4

Click Here

THE NEXT LEVEL §2.6 says, assume that the y's are (implicitly) a function of x.

$y = -1\left(x - \frac{3}{\sqrt{2}}\right) + \frac{3}{\sqrt{2}}$

Standard Example $x^2 + y^2 = 9$




$y^2 = 9 - x^2$

$y = \pm\sqrt{9 - x^2} \rightsquigarrow y = +\sqrt{9-x^2}$

$= (9-x^2)^{\frac{1}{2}} \Longrightarrow$

$y' = \frac{1}{2}(9-x^2)^{-\frac{1}{2}}(-2x) = \frac{-x}{\sqrt{9-x^2}}$

what's the slope @ $x = \frac{3}{\sqrt{2}}$?

$\nearrow \quad y'\left(\frac{3}{\sqrt{2}}\right) = \frac{-\frac{3}{\sqrt{2}}}{\sqrt{9 - \left(\frac{3}{\sqrt{2}}\right)^2}} = \frac{-\frac{3}{\sqrt{2}}}{\sqrt{9 - \frac{9}{2}}} = \frac{-\frac{3}{\sqrt{2}}}{\sqrt{\frac{9}{2}}} = \frac{-\frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}}} = -1$

§2.6 Says: Assume y is implicitly a function of x & find the slope on the circle of radius r=3 @ $(x,y) = \left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$:

Chain Rule for $\frac{d}{dx}\left[(y(x))^2\right] = 2(y(x)) \cdot y'(x)$

$x^2 + y^2 = 1$

$\frac{d}{dx}\left[x^2 + y^2 = 1\right] \Longrightarrow 2x + 2y \cdot y' = 0$

$\Longrightarrow 2yy' = -2x$

$y' = \frac{-2x}{2y} = -\frac{x}{y}$

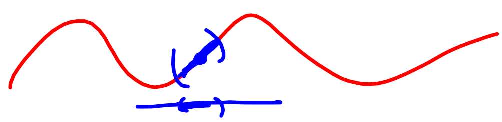
Plug in $(x,y) = \left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)$: $y'\Big|_{\left(\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}\right)} = -\frac{\frac{3}{\sqrt{2}}}{\frac{3}{\sqrt{2}}} = -1$ !

Same as when we

So Tangent Line Eq'n is

$y = m(x - x_1) + y_1$

$$\boxed{y = -1\left(x - \frac{3}{\sqrt{2}}\right) + \frac{3}{\sqrt{2}} = L(x)}$$ = Linearization



§2.4 Examples The temptation is to factor the coefficient of x out of the sine function:

$$\lim_{x\to 0} \frac{\sin(3x)}{x} = 3, \text{ but NOT the following way}$$

NOT!

$$\boxed{\frac{\sin(3x)}{x} = \frac{3\sin(x)}{x} = 3\,\frac{\sin(x)}{x} \xrightarrow{x\to 0} 3\cdot 1 = 3.}$$

BAD!

This is the proper way:

$$\frac{\sin(3x)}{x} = \frac{3}{3}\cdot\frac{\sin(3x)}{x} = \frac{3\sin(3x)}{3x} = 3\,\frac{\sin(3x)}{3x} \xrightarrow{x\to 0} 3\cdot 1 = 3$$

multiplied denominator by 3 to make it match the argument of sine upstairs, so I had to multiply the numerator by 3 to maintain equality.

$$\lim_{t\to 0} \frac{\tan(16t)}{\sin(4t)} = \frac{0}{0} \text{ is indeterminate!}$$

$$\boxed{\text{Leverage } \frac{\sin\theta}{\theta} \xrightarrow{\theta\to 0} 1}$$

$$\frac{\tan(16t)}{\sin(4t)} = \frac{\sin(16t)}{\cos(16t)}\cdot\frac{1}{\sin(4t)}\cdot\frac{t}{t}$$

$$= \frac{16}{16}\cdot\frac{\sin(16t)}{t}\cdot\frac{1}{\cos(16t)}\cdot\frac{t}{\sin(4t)}\cdot\frac{4}{4}$$

$$= \frac{16}{4}\cdot\frac{\sin(16t)}{16t}\cdot\frac{1}{\cos(16t)}\cdot\frac{4t}{\sin(4t)} \xrightarrow{t\to 0} 4\cdot 1\cdot 1\cdot 1 = 4$$

$$\frac{\tan(16t)}{\sin(4t)} = \frac{\sin(16t)}{\cos(16t)\ 16t}\cdot\frac{1\ 4t}{\sin(4t)}\cdot\frac{16}{4} \xrightarrow{t\to 0} \frac{1}{1}\cdot 1\cdot 4 = 4$$

§2.5 Challenging

$$\frac{d}{d\theta}\left[\cos^3(5\theta^2+2\theta)\cdot(3\theta^7+2\theta)\right]$$

$$\underbrace{\left(3\cos^2(5\theta^2+2\theta)\right)\left(-\sin(5\theta^2+2\theta)\right)(10\theta+2)}_{f'}\underbrace{(3\theta^7+2\theta)}_{g}$$

$$+\underbrace{\left(\cos^3(5\theta^2+2\theta)\right)}_{f}\underbrace{(21\theta^6+2)}_{g'}$$

Differentiating the $\cos^3(5\theta+2\theta)$:

$$\frac{d}{d\theta}\left[f(g(h(\theta)))\right]=\frac{d}{d\theta}\left[(g(h(\theta)))^3\right]$$

$$=\frac{df}{dg}\cdot\frac{dg}{dh}\cdot\frac{dh}{d\theta}=\left(3\cos^2(5\theta^2+2\theta)\right)\left(-\sin(5\theta^2+2\theta)\right)(10\theta+2)$$

$f(x)=x^3 \longrightarrow f(g)=g^3 \longrightarrow \frac{df}{dg}=3g^2$

$g(x)=\cos(x) \longrightarrow g(h)=\cos(h)$

$h(x)=5x^2+2x$