Normal to $y=3x-7$ thru $(1,-4)$

$m=3 \Rightarrow m_\perp = -\frac{1}{3}$

$y=-\frac{1}{3}(x-1)-4 = -\frac{1}{3}x+\frac{1}{3}-\frac{12}{3}$ ∴ $\boxed{y=-\frac{1}{3}x-\frac{11}{3}}$

§2.3 # 17 $f(x)=\frac{\sqrt{x}}{x+9}$ Quinton's version:

$f(x)=\frac{\sqrt{x}}{x+4}$ ∴ $(x_1,y_1)=(1,0.2)$

$\frac{\sqrt{1}}{5}=\frac{1}{5}=.2$

$f(x)=\frac{x^{\frac{1}{2}}}{x+4} \Rightarrow$

$f'(x)\Big|_{x=1} = \frac{\frac{1}{2}x^{-\frac{1}{2}}(x+4)-x^{\frac{1}{2}}(1)}{(x+4)^2}\Bigg|_{x=1} = \frac{\frac{1}{2}(5)-1}{5^2} = \frac{\frac{5}{2}-\frac{2}{2}}{25} = \frac{\frac{3}{2}}{\frac{25}{1}}$

$=\frac{3}{2}\cdot\frac{1}{25}=\frac{3}{50}=m_{tan} \Rightarrow y=\frac{3}{50}(x-1)+.2$ $\quad -3+10=7$

$=\frac{3}{50}x-\frac{3}{50}+\frac{2}{10}\cdot\frac{5}{5}=\frac{3}{50}x+\frac{7}{10}=y$

tan line

OR

$y=.6x+.7$

Normal Line

$y=-\frac{50}{3}(x-1)+.2$

$=-\frac{50}{3}x+\frac{50}{3}+\frac{2}{10}$

$\frac{5}{5}\cdot\frac{50}{3}+\frac{1}{5}\cdot\frac{3}{3}=\frac{250+3}{15}$

$=\frac{253}{15}$

$\boxed{y=-\frac{50}{3}x+\frac{253}{15}}$

#18 $h(x)=\frac{x^2}{8x+1}$ $\quad$ $\frac{x^2}{4x+7}$ ($f \to x^2$, $g \to 4x+7$) for questioner.

$\Rightarrow h'(x)=\frac{f'g-fg'}{g^2}=\frac{2x(4x+7)-x^2(4)}{(4x+7)^2}=\frac{8x^2+14x-4x^2}{(4x+7)^2}$

$=\frac{4x^2+14x}{(4x+7)^2} \Rightarrow h''(x)=\frac{(8x+14)(4x+7)^2-(4x^2+14x)(2(4x+7)^1(4))}{(4x+7)^{2\to 4}}$

I'm cheating here b/c you don't "know" chain Rule, yet.

$g(x)=(4x+7)^2 \Rightarrow g'(x)=2(4x+7)^1(4)$

general power rule, which is a special case of the Chain Rule

$\frac{d}{dx}[f(g(x))]$

$=\frac{df}{dg}\cdot\frac{dg}{dx}$

$\frac{d}{dx}\left[(g(x))^n\right]=n\,g(x)^{n-1}\cdot g'(x)$

$=n\,g(x)^{n-1}\cdot\frac{dg}{dx}$

$h(x) = \frac{x^2}{4x+7}$ for questioner.

$h'(x) = \frac{f'g - fg'}{g^2} = \frac{4x+7) - x^2(4)}{(4x+7)^2} = \frac{8x^2 + 14x - 4x^2}{(4x+7)^2} = \frac{4x^2+14x}{(4x+7)^2}$

$= 2\left(\frac{2x^2+7x}{(4x+7)^2}\right)$

$h''(x) = \left[\frac{(4x+7)(4x+7)^2 - (2x^2+7x)(2)(4x+7)(4)}{(4x+7)^4}\right](2)$

$= \left(\frac{(4x+7)[(4x+7)^2 - 16x^2 - 14x]}{(4x+7)^4}\right)(2)$ $\quad 14x \to 56x$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$= \left(\frac{\cancel{16x^2} + 56x + 49 - \cancel{16x^2} - 14x}{(4x+7)^3}\right)(2) = \left(\frac{49}{(4x+7)^3}\right)(2)$ $\quad 14x \to 56x$

$= \frac{98}{(4x+7)^3} = h''(x)$ if I didn't blunder.

$h''(x) = 98(4x+7)^{-3}$

$h'''(x) = -3(98)(4x+7)^{-4}(4)$

$h^{(iv)}(x) = -12(98)(-4)(4x+7)^{-5}(4)$

$h^{(v)}(x) = 12(16)(98)(-5(4x+7)^{-6}(4))$

General Power Rule Galore!

#25 Find eq'n of normal line to $f(x)=\sqrt{x}$ that is $\parallel$ to $4x+y=1$

$\boxed{m=-4}$

$Ax+By=C \Rightarrow m=-\frac{A}{B}$

$By=-Ax+C$

$y=-\frac{A}{B}x+C$

$f(x)=\sqrt{x}=x^{\frac{1}{2}} \Longrightarrow$

$f'(x)=\frac{1}{2}x^{-\frac{1}{2}}=m$

Want $m_\perp=-4$, so we want $m_{tan}=\frac{1}{4}$

Set $\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{4}$

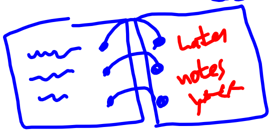
$x^{-\frac{1}{2}}=\frac{1}{2}$

$x^{\frac{1}{2}}=2$

$x=4$

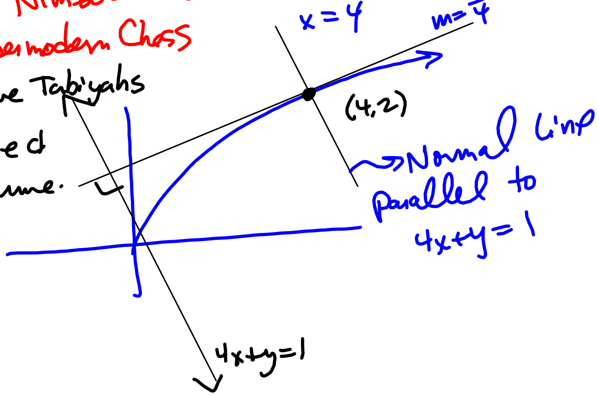
$f'(4)=\frac{1}{2}(4)^{-\frac{1}{2}}=\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{4}$

$\Rightarrow m_\perp$ @ $x=4$ is $m_\perp=-4$



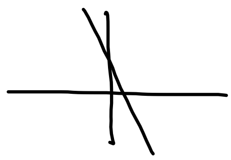
Aron Nimzowitsch
Hypermodern Chess

20-move Tabiyahs ruined the game.



$4x+y=1$

$y=-4x+1$



To find a spot where the normal line to $f(x)=\sqrt{x}$ was parallel to $4x+y=1$, I found where the tangent to $f(x)$ was $\perp$ to $4x+y=1$

so I did $f'(x)\overset{SET}{=}\frac{1}{4}$ = negative reciprocal of -4.

Now build the eq'n of the line:

$y=m(x-x_1)+y_1 = \frac{1}{4}(x-4)+2$

$= \frac{1}{4}x-1+2 = \boxed{\frac{1}{4}x+1=y}$

This is ~~tangent~~ line!

They want normal line!

$y=-4(x-4)+2 = -4x+16+2$

$\boxed{y=-4x+18}$

we knew we needed to find $x \ni f'(x)=\frac{1}{4}$.

$P(2) = 10$ $\quad P(x) = ax^2 + bx + c$

$P'(2) = 12$ $\quad P'(x) = 2ax + b$

$P''(2) = 8$ $\quad P''(x) = 2a \overset{SET}{=} 8 \rightarrow a = 4$

$P(x) = 4x^2 - 4x + 2$ $\quad P(x) = 4x^2 + bx + c$

$P(2) = \checkmark$ $\quad P'(x) = 8x + b$

$P'(2) = 8x - 4 \Big|_{x=2} = 16 - 4 = 12 \checkmark$ $\quad P'(2) = 16 + b \overset{SET}{=} 12$

$b = -4$

$P(2) = 4(2)^2 - 4(2) + c = 10$

$P''(2) = 8 \checkmark$ $\quad 16 - 8 + c = 8 + c = 10 \rightarrow$

$c = 2$

Same #25 only $6x+y=1$ $\perp$ to tangent to $y=\sqrt{x}$

$m_{\perp} = -6 \Longrightarrow m_{tan} = \frac{1}{6} \overset{SET}{=} \frac{1}{2\sqrt{x}} \Longrightarrow$ $(9,3)$

$\to \sqrt{x}$

$$2\sqrt{x} = 6$$
$$\sqrt{x} = 3$$
$$x = 9$$

$(x_1, y_1) = (9,3)$

$y = m(x-x_1)+y_1$

$= -6(x-9)+3$

$$y = -6(x-9)+3$$
$$= -6x+54+3 = \boxed{-6x+57 = y}$$

Section 2.4 - Derivatives of Trig Functions.

I think the proof in the videos is pretty good.

Difference Quotient for $\sin(x)$:

Recall $\sin(x+y) = \sin(x)\cos(y) + \sin(y)\cos(x)$

$$\frac{\sin(x+h) - \sin(x)}{h} = \frac{\sin(x)\cos(h) + \sin(h)\cos(x) - \sin(x)}{h}$$

$$= \frac{\sin(x)\cos(h) - \sin(x) + \sin(h)\cos(x)}{h}$$

$$= \frac{\sin(x)(\cos(h) - 1)}{h} + \frac{\sin(h)\cos(x)}{h}$$

$$= \frac{\cos(h) - 1}{h}\sin(x) + \frac{\sin(h)}{h}\cos(x) \xrightarrow{h \to 0} \cos(x)$$

0 as $h \to 0$ 1 as $h \to 0$ The goal!

$$\theta > \sin\theta \longrightarrow \frac{\sin\theta}{\theta} < 1$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta > \theta$$

$$\frac{\sin\theta}{\theta} > \cos\theta$$

$$\cos\theta < \frac{\sin\theta}{\theta} < 1$$

$\theta \to 0$ $\theta \to 0$ $\theta \to 0$

$$1 \le \lim_{\theta \to 0} \frac{\sin\theta}{\theta} \le 1$$

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$$

https://harryzaims.com/201/videos/chapter-02/2-4/videos/00a-derivative-of-sine-proved.mp4

Once you have $\frac{d}{dx}[\sin(x)]$ the rest flow quickly

https://harryzaims.com/201/videos/chapter-02/2-4/videos/00-derive-derivatives-of-OTHER-trig-functions_First_Frame.png

$$\frac{d}{dx}[\sin(x)] = \cos(x)$$

$$\frac{d}{dx}[\cos(x)] = -\sin(x)$$

$$\frac{d}{dx}[\tan(x)] = \frac{d}{dx}\left[\frac{\sin(x)}{\cos(x)}\right] = \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} = \boxed{\sec^2(x)}$$