Questions?

The "worst" 2.2 exercise: $f(x)=x|x| = \begin{cases} x\cdot x = x^2 & \text{if } x\ge 0 \\ x\cdot(-x) = -x^2 & \text{if } x<0\end{cases}$

2.2 method:

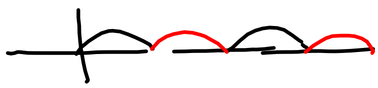
$$\frac{(x+h)^2-x^2}{h} = \frac{x^2+2xh+h^2-x^2}{h} = \frac{2xh+h^2}{h} = \frac{h(2x+h)}{h} = \underset{(h\neq 0)}{2x+h} \xrightarrow{h\to 0} 2x \quad \text{for } x\ge 0$$

$$\frac{-(x+h)^2-(-x^2)}{h} = \frac{-(x^2+2xh+h^2)+x^2}{h} = \frac{-x^2-2xh-h^2+x^2}{h}$$

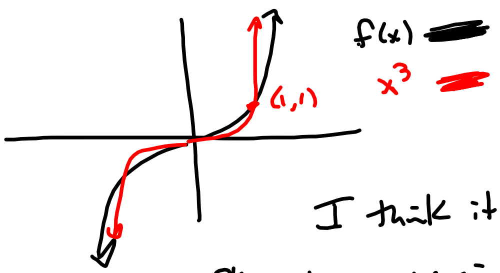
$$= \frac{-2xh-h^2}{h} = \frac{h(-2x-h)}{h} \underset{(h\neq 0)}{=} -2x-h \xrightarrow{h\to 0} -2x = f'(x),\ x<0$$

$$f'(x) = \begin{cases} 2x & \text{if } x\ge 0 \\ -2x & \text{if } x<0\end{cases} = 2|x|$$

$|\sin(x)|$



$$f(x)=x|x| = \begin{cases} x^2 & \text{if } x\ge 0 \\ -x^2 & \text{if } x<0\end{cases}$$



I think it's difb^{le} Everywhere.

$f'(x)=2|x|$, converging smoothly @ $x=0$ to $f'(0)=0$.

f' is not difb^{le} @ $x=0$, because it has a cusp (sharp point) @ $x=0$, so no f'' @ $x=0$.

Recall last time $(fg)' = f'g + fg'$

$\boxed{\text{Proof}}$ $\dfrac{f(x+h)g(x+h) - f(x)g(x)}{h} =$

The trick

$$= \frac{f(x+h)g(x+h) \overbrace{- f(x)g(x+h) + f(x)g(x+h)} - f(x)g(x)}{h}$$

Quotient Rule: $\boxed{\left(\dfrac{f}{g}\right)' = \dfrac{f'g - fg'}{g^2}}$

Sketch of Proof.

$$\frac{\frac{f(x+h)}{g(x+h)} - \frac{f(x)}{g(x)}}{h} = \frac{1}{h}\left[\frac{f(x+h)g(x) - f(x)g(x+h)}{g(x+h)g(x)}\right]$$

$$= \frac{1}{h}\left[\frac{\overbrace{f(x+h)g(x) - f(x)g(x)}^{f'g} + \overbrace{f(x)g(x) - f(x)g(x+h)}^{\to -fg'}}{\underbrace{g(x+h)g(x)}_{\to g^2}}\right]$$

Find the Derivative: §2.3

$h(x) = (x^3+2x^2-5)(7x^2+x) = fg \Longrightarrow$

$h'(x) = f'g + fg' = (3x^2+4x)(7x^2+x) + (x^3+2x^2-5)(14x+1)$

Good! But WebAssign always wants this crap expanded! UGH!

$= 21x^4 + 3x^3 + 28x^3 + 4x^2 + 14x^4 + 28x^3 - 5 + x^3 + 2x^2 - 5$ Terrible

$= 35x^4 + 60x^3 + 6x^2 - 10 = h'(x)$

$= 21x^4 + 3x^3 + 28x^3 + 4x^2 + 14x^4 + 28x^3 - 70x + x^3 + 2x^2 - 5$

$= 35x^4 + 60x^3 + 6x^2 - 70x - 5$

What did we learn?

Being in a hurry slows you down.

Let's check that work:

$expand(h(x))$

$$7x^5 + 15x^4 + 2x^3 - 35x^2 - 5x$$

$diff(\%, x)$

$$35x^4 + 60x^3 + 6x^2 - 70x - 5$$

Quotients are very messy.

Perfect for written work!

$$h(x) = \frac{x^2+2}{x^2-2} = \frac{f}{g} \to \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} = \boxed{\frac{2x(x^2-2) - (x^2+2)(2x)}{(x^2-2)^2}}$$

$f(x) = x^2+2$ $\quad$ $g(x) = x^2-2$

$f'(x) = 2x$ $\quad$ $g'(x) = 2x$

Extra writing to be more secure as you start out.

WRITE MUCH; THINK LITTLE.

Machine

GODLIKE, THINKING SYMBOLICALLY / BIG-PICTURE

$$= \frac{2x^3 - 4x - 2x^3 - 4x}{(x^2-2)^2} = \boxed{\frac{-8x}{x^4 - 4x^2 + 4}}$$

WEBASSIGN MIGHT REQUIRE THIS FORM. UGH!

#13 §2.3

$$\frac{y}{y+\frac{b}{y}} = h(y)$$

$$h'(y) = \frac{1\left(y+\frac{b}{y}\right) - y\left(1-\frac{b}{y^2}\right)}{\left(y+\frac{b}{y}\right)^2}$$

$r(y) = \frac{b}{y} \Rightarrow$ Harder

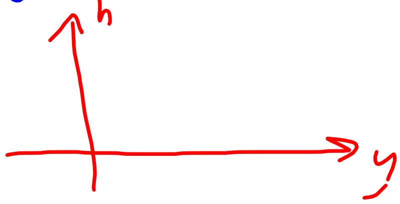
$$r'(y) = \frac{0 \cdot y - b \cdot 1}{y^2} = -\frac{b}{y^2}$$

$\boxed{\text{But}}$ $\frac{b}{y} = by^{-1} \Rightarrow$ Easier using power Rule.

$$r'(y) = -by^{-2} = -\frac{b}{y^2}$$

$$h(y) = \frac{y}{y+\frac{b}{y}} = \frac{y}{y+by^{-1}} = \frac{f}{g} \Rightarrow h'(y) = \left(\frac{f}{g}\right)'$$

$$= \frac{f'g - fg'}{g^2}$$



$f = y \Rightarrow f' = 1$

$g = y + by^{-1} \Rightarrow g' = 1 - by^{-2}$

The 'y' is the independent variable, here, which can throw you.

In this one, it is clear from the context that 'y' is the independent variable, and the variable with respect to which we are differentiating.

$f(\theta) = xy\theta + 27\theta^2$

$\Rightarrow f'(\theta) = xy + 54\theta$

Everything BUT θ is treated as a constant.

We differentiate wrt θ.

$3x+2=5$	$3y+2=5$
$3x=3$	$3y=3$
$x=1$	$y=1$

§2.3 #12

$$f(t) = \frac{\sqrt[3]{t}}{t-9} = \frac{t^{1/3}}{t-9} \Longrightarrow$$

$$\boxed{f'(t) = \frac{\frac{1}{3}t^{-\frac{2}{3}}(t-9) - t^{\frac{1}{3}}(1)}{(t-9)^2}} = \frac{\frac{1}{3}t^{-\frac{2}{3}+1} - \frac{1}{3}t^{-\frac{2}{3}}\cdot 9 - t^{\frac{1}{3}}}{(t-9)^2}$$

Stop here

for mills

$$= \frac{\frac{1}{3}t^{\frac{1}{3}} - 3t^{-\frac{2}{3}} - t^{\frac{1}{3}}}{(t-9)^2} = \frac{-\frac{2}{3}t^{\frac{1}{3}} - 3t^{-\frac{2}{3}}}{(t-9)^2} = \frac{\frac{-2t^{\frac{1}{3}}}{3}\cdot\frac{t^{\frac{2}{3}}}{t^{\frac{2}{3}}} - \frac{3}{t^{2/3}}\cdot\frac{3}{3}}{(t-9)^2}$$

$$= \frac{\frac{-2t-9}{3t^{\frac{2}{3}}}}{\frac{(t-9)^2}{1}} = \frac{-2t-9}{3t^{\frac{2}{3}}}\cdot\frac{1}{(t-9)^2}$$

$$= \frac{-2t-9}{3t^{\frac{2}{3}}(t-9)^2}$$ is what WebAssign wants.

$2/3 < \frac{2}{3}$

BAD BETTER

For Hard Chargers (Planting Seeds)

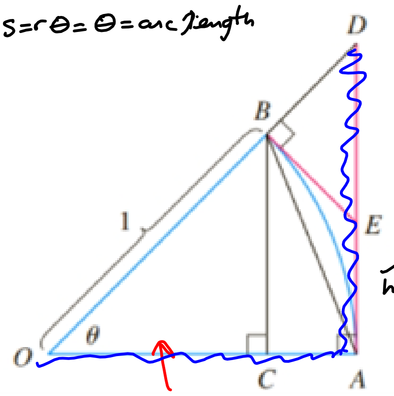
$\frac{d}{dx}[\sin(x)] = \cos(x)$ — Next time: Big Proof

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(\theta)] = 0$ TRICKED Ya!

$\frac{d}{dx}[\tan(x)] = \sec^2(x)$

$s = r\theta = \theta =$ arc length



(I) $\sin\theta < \theta$ so $\frac{\sin\theta}{\theta} < 1$

(II) $\cos\theta < \frac{\sin\theta}{\theta} < 1$

Then squeeze this limit:

$\lim_{h \to 0} \frac{\sin(\theta+h) - \sin(\theta)}{h}$

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§2.5 $(f(g(x)))' = (f(\square))' = f'(\square)\cdot\square'(x)$

$\frac{d}{dx}[f(g(x))] = \frac{df}{dg}\cdot\frac{dg}{dx}$

$f(x) = x^2 \Rightarrow f'(x) = 2x$

$f(\square) = \square^2 \Rightarrow f'(\square) = 2\square$

$h(x) = (x^2+3x)^2 = f(g(x))$

Good 4 mills.

$\left.\begin{aligned} f(g) &= g^2 \\ g(x) &= x^2+3x \end{aligned}\right\} \frac{df}{dg}\cdot\frac{dg}{dx} = (2g)\cdot(2x+3) = \boxed{2(x^2+3x)\cdot(2x+3)}$

$(x^2+3x)^2 = x^4 + 2x^2\cdot 3x + 9x^2 = x^4 + 6x^3 + 9x^2$

$\Rightarrow h'(x) = 4x^3 + 18x^2 + 18x$

$\not{d}\ h'(x) = 2(2x^3 + 3x^2 + 6x^2 + 9x) = 2(2x^3 + 9x^2 + 9x)$

$= 4x^3 + 18x^2 + 18x$

TRAILBLAZER LESSONS

$h(x) = (x^2 - 5x + 2)^{33}$

$\Rightarrow h'(x) = 33(x^2-5x+2)^{32}\cdot(2x-5)$ Beautiful!