Today: Some are done w/ §2.1. I still have some tricks for it & §2.2.

§2.1 – Slope @ $(a, f(a))$ is $f'(a) = m_{tan} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$
= Derivative of f @ $x = a$.

Tangent Line to $f(x)$ @ $(a, f(a))$ is $y = f'(a)(x-a) + f(a)$

$m, (x_1, y_1) \Rightarrow y = f(x) = m(x - x_1) + y_1$ $(x_1, y_1)$ $y = m_{tan}(x - x_1) + y_1$

§2.2 – Derivative Function, $f'(x)$, where we let a vary

So The derivative of $f(x)$ is the function of x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ , provided the limit exists.

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Checking your "Derivatives by the limit definition" by $\boxed{\text{§2.3 CHEATS.}}$

POWER RULE: $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$

Liebniz Notation: $\frac{d}{dx}[x^n] = nx^{n-1}$

$\boxed{E}$ $\frac{d}{dx}[x^5] = 5x^4$. $\frac{d}{dx}$ = "derivative $\overbrace{\text{with respect}}^{\text{wrt}}$ to x of..."

SUM RULE: $(f+g)' = f' + g'$ OR $\frac{d}{dx}[f+g] = \frac{df}{dx} + \frac{dg}{dx}$

CONSTANT MULTIPLE RULE: $(af + bg)' = af' + bg'$, where a & b are numbers (scalars) & f & g are funcs.

$\lim_{x \to 3}(7x) = 7 \lim_{x \to 3}(x) = 7(3) = 21$

LIEBNIZ VERSION: $\frac{d}{dx}[af + bg] = a\frac{df}{dx} + b\frac{dg}{dx}$

This means "$\frac{d}{dx}$ is a linear operator."

It's a consequence of properties of limits & the fact that derivatives are limits.

The derivative respects addition and scalar (constant) multiplication.

$af + bg$ is a linear combination of f & g

**EXAMPLE** Evaluate $\frac{d}{dx}\left[x^7 - x^1\right]$

$\frac{d}{dx}\left[x^1\right] = 1x^0 = 1$

CHEAT: $= 7x^6 - 1 = f'(x)$.

OLD-SCHOOL: I want to refresh you on ALGEBRA's

BINOMIAL THEOREM:

RECALL Binomial Coefficients

$$(x+h)^n = x^n + \binom{n}{1}x^{n-1}h^1 + \binom{n}{2}x^{n-2}h^2 + \cdots + \binom{n}{n-2}x^2h^{n-2} + \binom{n}{n-1}xh^{n-1} + h^n$$

where $\binom{n}{k} = {}_nC_k = C(n,k) = \frac{n!}{(n-k)!\,k!}$ = Combinations of $n$ things taken $k$ at a time = $\boxed{\text{"n choose k."}}$

= the # of distinct subsets of $k$ elements taken from a set of $n$ elements.

$$7 \text{ choose } 3 = \binom{7}{3} = \frac{7!}{4!\,3!} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{(4\cdot3\cdot2\cdot1)(3\cdot2\cdot1)} = \frac{7\cdot6\cdot5}{3\cdot2\cdot1} = 7\cdot5 = 35$$

PASCAL'S TRIANGLE

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1 ← $(x+h)^7$ row!

$C(7,3) = \binom{7}{3}$

$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

S 2.1 #3 in my notes was $x - x^7$

$$(x+h)^7 = x^7 + 7x^6h + 21x^5h^2 + 35x^4h^3 + 35x^3h^4 + 21x^2h^5 + 7xh^6 + h^7 \,!$$

So 2.1 wants $f'(1)$: for $f(x) = x^7 - x$

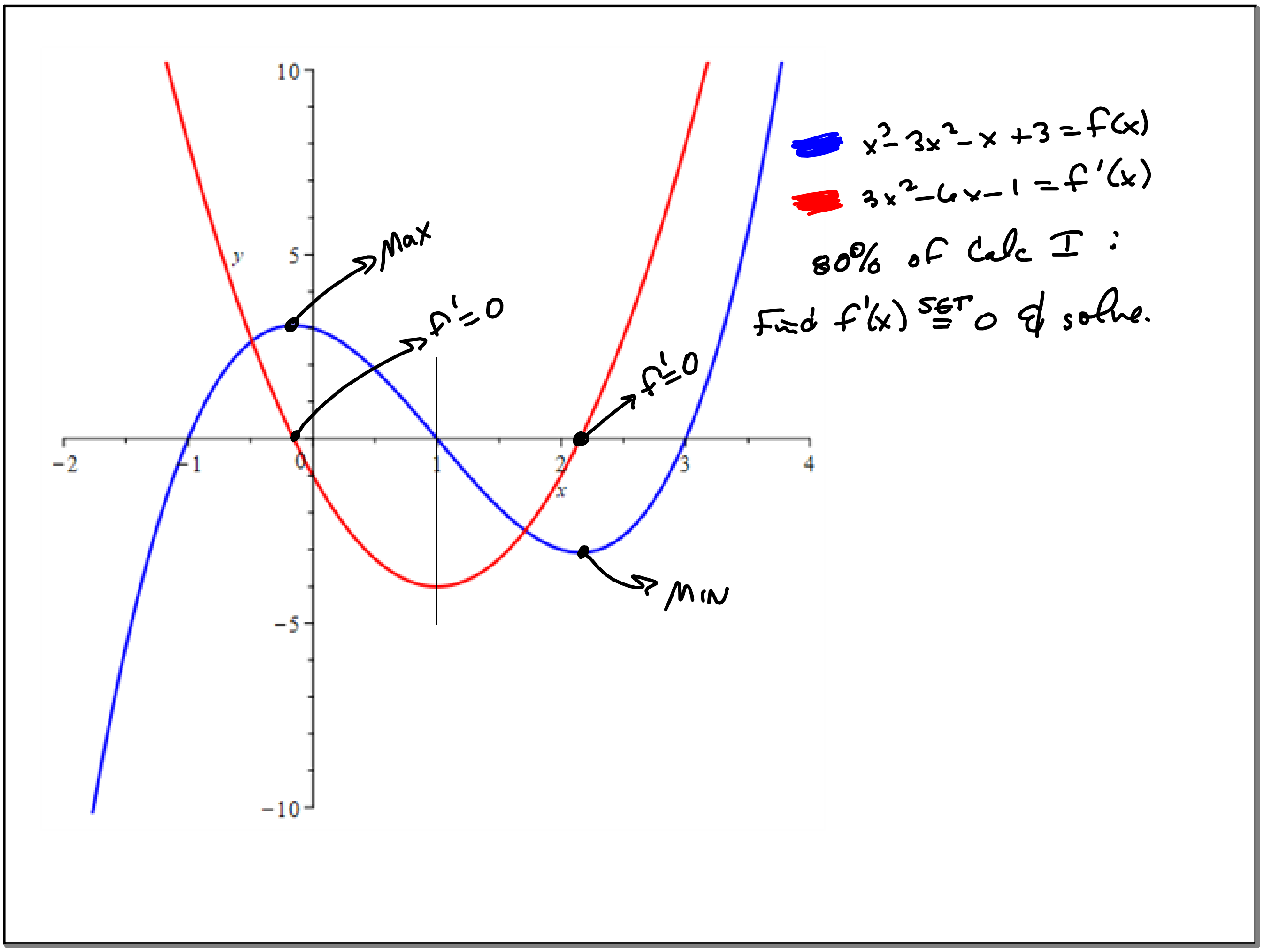
$$\frac{f(1+h) - f(1)}{h} =$$

$$= \frac{1 + 7h + 21h^2 + 35h^3 + 35h^4 + 21h^5 + 7h^6 + h^7 - (1+h) - (1^7 - 1)}{h}$$

$$= \frac{1 + 7h + 21h^2 + 35h^3 + 35h^4 + 21h^5 + 7h^6 + h^7 - 1 - h}{h}$$

$$= \frac{\cancel{h}(7 + 21h + 35h^2 + 35h^3 + 21h^4 + 7h^5 + h^6 - 1)}{\cancel{h}}$$

$\xrightarrow{h \to 0}$ $7 - 1 = 6$ = slope of $f(x) = x^7 - x$ @ $x = 1$.



x³−3x²−x+3 = f(x)
3x²−6x−1 = f'(x)
80% of Calc I :
Find f'(x) SET = 0 & solve.
Max
f' = 0
f' = 0
MIN

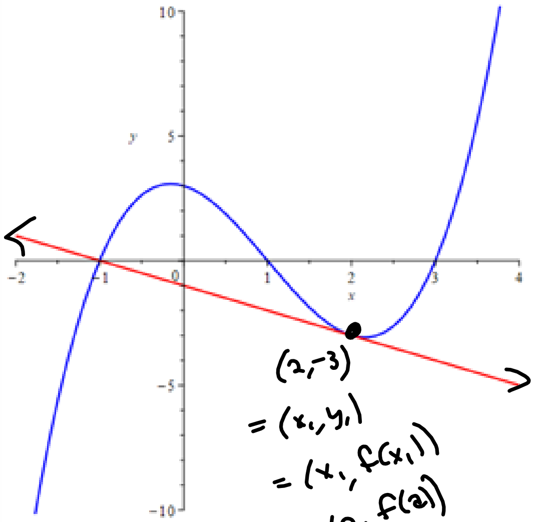
$\binom{4}{2} = C(4,2) = \frac{4!}{2!\,2!} = \frac{4\cdot 3}{2\cdot 1} = 6$

$\{a, b, c, d\}$

$\{a,b\}, \{a,c\}, \{a,d\},$ 6 of 'em

$\{b,c\}, \{b,d\}, \{c,d\}$

$(1)(-1)(3) = -3$



Graph of $f(x) = (x-1)(x-3)(x+1)$

$x^3 - 3x^2 - x + 3 = f(x)$

& its tangent line at $(2,-3)$

$(2,-3)$
$= (x_1, y_1)$
$= (x_1, f(x_1))$
$= (2, f(2))$

2.3 way:

$f'(x) = 3x^2 - 6x - 1$

$f'(2) = 3(2)^2 - 6(2) - 1$

$= 3(4) - 12 - 1$

$= 12 - 13 = -1 = m_{tan}$

$y = f'(2)(x-2) + f(2)$

$= -1(x-2) + (-3)$

$y = -(x-2) - 3$ steve likes

$= -x + 2 - 3 = -x - 1$ webAssign likes.

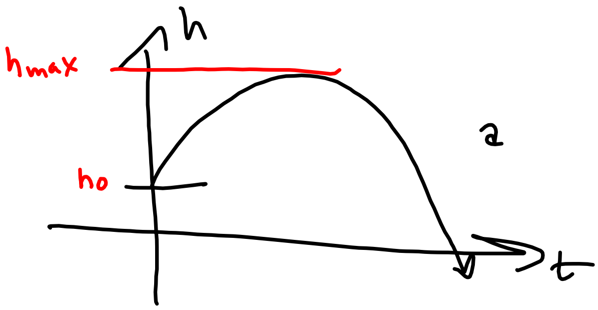
FALLING BODY Newton's Apple.

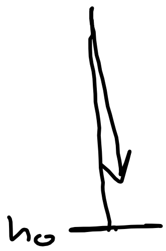
height $= h = h(t) = \frac{1}{2}at^2 + v_0 t + h_0$, where

$a =$ acceleration of gravity ($-32 \frac{ft}{s^2}$ OR $-9.8 \frac{m}{s^2}$)

$v_0 =$ initial velocity

$h_0 =$ initial height




How fast?

$h'(t) = at + v_0$

what's its acceleration?

$h''(t) = a$

Distance falling from top of building:

$4.9t^2$ (from §1.4)

$= \frac{1}{2}gt^2$ in MKS system (metric)

$a, b$ constants, like $3, 7$. $f, g$ functions, like $f(x) = x^2 + 1$, $g(x) = 2x^3 - x$

$(3f + 7g)' = 3f' + 7g'$ $\quad \frac{d}{dx}$ is linear operator.

2.3 Product Rule $(fg)' = f'g + fg'$

Book version can cause Dain Bramage.

Quotient Rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

Proof of Product Rule:

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h}$$

$$= \frac{\big(f(x+h) - f(x)\big)g(x) + f(x)\big(g(x+h) - g(x)\big)}{h}$$

$$= \frac{\big(f(x+h) - f(x)\big)g(x)}{h} + \frac{f(x)\big(g(x+h) - g(x)\big)}{h}$$

$$= \left(\frac{f(x+h) - f(x)}{h}\right)g(x) + f(x)\left(\frac{g(x+h) - g(x)}{h}\right)$$

If $f'(x)$ & $g'(x)$ exist, then I can do this:

$$\xrightarrow{h \to 0} f'(x)g(x) + f(x)g'(x) = f'g + fg' \quad \blacksquare$$

§2.4 Derivatives of trig functions.

§2.5 Chain Rule.

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg} \cdot \frac{dg}{dx}$$

$$\frac{d}{dx}\left[\left(x^5\right)^7\right] = 7\left(x^5\right)^6\left(5x^4\right) = 7x^{30} \cdot 5x^4 = 35x^{34}$$

$$\frac{d}{dx}\left[\left(x^5\right)^7\right] = \frac{d}{dx}\left[x^{35}\right] = 35x^{34}$$

$f(x) = -x^3$

$-f(x) = x^3$ & use $(-f)' = -(f)'$

$$\frac{f(a+h)-f(a)}{h} = \frac{-(a+h)^3-(-a^3)}{h}$$

$$= -\frac{(a^3+3a^2h+3ah^2+h^3)+a^3}{h}$$

$$= \frac{-a^3-3a^2h-3ah^2-h^3+a^3}{h}$$

$$= \frac{-3a^2h-3ah^2-h^3}{h} = \frac{h(-3a^2-3ah-h^2)}{h}$$

$$= -3a^2-3ah-h^2 \xrightarrow{h \to 0} \boxed{-3a^2}$$

$(h \neq 0)$