0/4 points SCalc9 1.7.035.

A graphing calculator is recommended.

(a) For the limit $\lim_{x\to 1}(x^3+x+3)=5$, use a graph to find a value of $\delta$ that corresponds to $\varepsilon = 0.5$. (Round your answer down to three decimal places.)

$\delta =$ ✗ 0.114

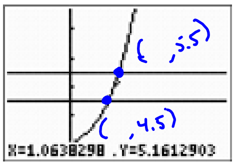
(b) By solving the cubic equation $x^3+x+3=5+\varepsilon$, find the largest possible value of $\delta$ that works for any given $\varepsilon > 0$.

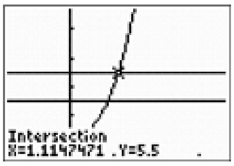
$\delta(\varepsilon) =$ ✗ $\dfrac{(216+108\varepsilon+12\sqrt{336+324\varepsilon+81\varepsilon^2})^{2/3}-12}{6(216+108\varepsilon+12\sqrt{336+324\varepsilon+81\varepsilon^2})^{1/3}}-1$

(c) Put $\varepsilon = 0.5$ in your answer to part (b). (Round your answer down to three decimal places.)

$\delta(0.5) =$ ✗ 0.114

2




$x \approx 1.114747\,1$

$.114747\,1$

$.114$

Claim: $\lim_{x\to 1}(x^3+x+3)=5$

Scratch: $|x^3+x+3-5| < \varepsilon$ we want

$x^3+x-2$ has a factor of $x-1$ in it.

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

This says DOH!

$x^3+x-2 = (x-1)\underbrace{(x^2+x+2)}_{\text{Need a handle (bound) on this}}$ want $< \varepsilon$

$(x-1) \to\ < \delta$

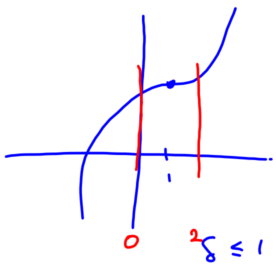
Assume $\delta$ is small, say, $\delta \le 1$.

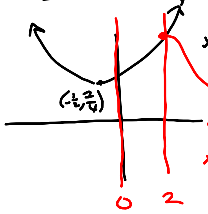
Let's see how BIG

$x^2+x+2$

$x^2+x+\left(\frac{1}{2}\right)^2-\frac{1}{4}+2$

$= \left(x+\frac{1}{2}\right)^2+\frac{7}{4}$




$2^2+2+2=8$

So on $[0,2]$ $x^2+x+2 \le 8$

Look: $\delta \le 1$. Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$. Then

$0 < |x-1| < \delta \Rightarrow$

$|(x^3+x+3)-5| = |x^3+x-2| = |x-1||x^2+x+2|$

$< \delta \cdot 8 \le \frac{\varepsilon}{8}\cdot 8 = \varepsilon$ ∎

---

Requires the assumption that $\delta \le 1$, which is no handicap.

Claim $\lim_{x\to 1}(x^3+x+3)=5$ $\quad x^3+x+3 \xrightarrow{x\to 1} 5$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\{1, \frac{\varepsilon}{8}\}$. Then if $0 < |x-1| < \delta$, we have $|x^3+x+3-5| = |x^3+x-2|$ $= |x-1||x^2+x+2| < \delta\cdot 8 \le \frac{\varepsilon}{8}\cdot 8 = \varepsilon$ ∎

$$\frac{\left(216+108\epsilon+12\sqrt{81\epsilon^2+324\epsilon+336}\right)^{1/3}}{6} - \frac{2}{\left(216+108\epsilon+12\sqrt{81\epsilon^2+324\epsilon+336}\right)^{1/3}}$$

is one solution.

https://math.vanderbilt.edu/schectex/courses/cubic/

So what Jocelyn's saying is that 1.7 #19 really sucks.

## §1.7 The Basic Proof

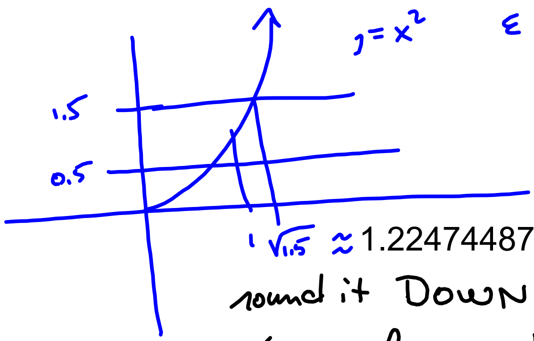
Prove $\lim_{x \to 3} (7x+2) = 23$

**Proof** Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{7}$. Then

$$0 < |x-3| < \delta \Rightarrow |7x+2-23| = |7x-21| = 7|x-3|$$

$$< 7\delta = 7 \cdot \frac{\varepsilon}{7} = \varepsilon \blacksquare$$

$\varepsilon = .5$, $x = 1$




1 $\sqrt{1.5} \approx$ 1.224744871, so, to stay INSIDE,
round it DOWN to 1.224
(ordinarily round to 1.225)
Victoria points out that 1.224 corresponds to $\delta$-value of 0.224.

# Quick Quadratic Proof of Limit.

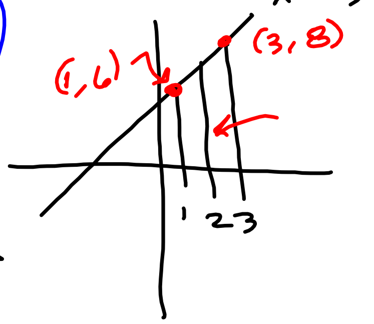
Claim: $\lim_{x\to 2}(x^2+3x-7)=3$

Scratch $|x^2+3x-7-3| = |x^2+3x-10| = \underbrace{|x-2|}_{<\delta}\underbrace{|x+5|}_{\text{Bound on } |x+5|}$ Want $<\varepsilon$

Assume $x$ is pretty close to 2, say

$|x-2| \le 1$ (i.e., $\delta \le 1$)

$$\begin{array}{ccccc} -1 & \le & x-2 & \le & 1 \\ +7 & = & +7 & = & +7 \\ \hline 6 & \le & x+5 & \le & 8. \end{array}$$




Proof Let $\varepsilon>0$ be given. Define $\delta=\min\{1, \frac{\varepsilon}{8}\}$. Then

$0<|x-2|<\delta \implies |x^2+3x-7-3| = |x^2+3x-10|$

$= |x+5|\underbrace{|x-2|}_{\delta} < 8|x-2| < 8\delta \le 8\cdot\frac{\varepsilon}{8} = \varepsilon$ ■

Steve's arguing w/ himself about "<" vs "≤." Idiot.

Shorthand

$A \Rightarrow B$ A implies B

$A \Leftrightarrow B$ } A if and only if B.
A iff B }

$\forall$ For all, for each or for every

$\exists$ There is, there exists

I think, $\therefore$ I $\exists$.

$\therefore$ therefore

Ⓦ otherwise, else

b/c because.

cnt$^{s}$ $\Leftrightarrow$ continuous (No holes/breaks)

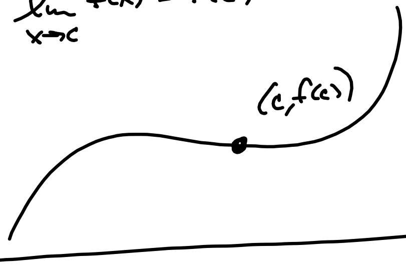
differentiable $\Leftrightarrow$ difb$^{\ell}$ (has a derivative)

$\mathbb{N}$ Naturals, $\mathbb{R}$ Reals, $\mathbb{Q}$ Rationals

$\mathbb{Z}$ Integers.

$f$ is cnt's @ $x=c$ if

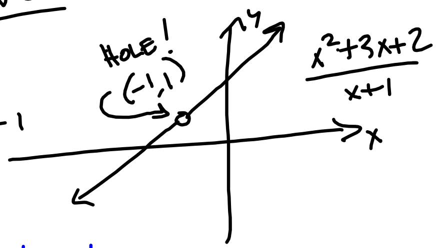
$\lim_{x\to c} f(x) = f(c)$




Non examples

$\frac{x^2+3x+2}{x+1}$ is NOT cnt's @ $x=-1$

$= \frac{(x+2)\cancel{(x+1)}}{\cancel{x+1}} = x+2$, $x \neq -1$



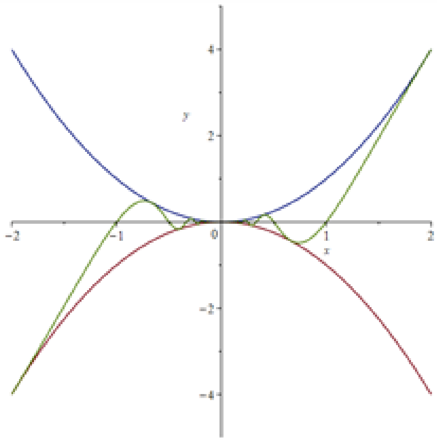

This is a case of a removable discontinuity.

Just define $f(-1) = 1$.

$$f(x) = \begin{cases} \frac{x^2+3x+2}{x+1} & \text{if } x \neq -1 \\ 1 & \text{if } x = -1 \end{cases}$$

more elaborate:

$\lim_{x\to 0} \left(x^2 \sin\left(\frac{\pi}{x}\right)\right) = 0$



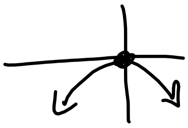
$-1 \leq \sin\left(\frac{\pi}{x}\right) \leq 1$

$-x^2 \leq x^2 \sin\left(\frac{\pi}{x}\right) \leq x^2$

$\downarrow_{x\to 0}$ 0 $\quad$ $\downarrow_{x\to 0}$ 0 $\quad$ $\downarrow_{x\to 0}$ 0

So, by Squeeze Theorem

$x^2 \sin\left(\frac{\pi}{x}\right) \xrightarrow{x\to 0} 0$



We build a function that's cnt's $\forall x \in \mathbb{R}$.

Define $g(x) = \begin{cases} x^2 \sin\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$