§1.3 # 17

$f(x) = \frac{x}{1+x}$, $g(x) = \sin(4x)$

ıd the function $f \circ g$.

$\circ\, g)(x) =$ ✗ $\frac{\sin(4x)}{\sin(4x)+1}$

ate the domain of the function. (Let n represent an arbitrary integer.)

$\mathcal{D}(f\circ g)$:

$\sin(4x) = -1$

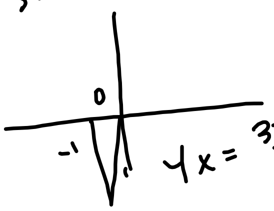
Need $\sin(4x) + 1 \neq 0$

$\sin(4x) \neq -1$

$4x \in [0, 2\pi]$

$0 \le 4x \le 2\pi$

$0 \le x \le \frac{\pi}{2}$



$4x = \frac{3\pi}{2}$

$x = \frac{3\pi}{8}$

$\{x \mid x = \frac{3\pi}{8} + 2n\pi \mid n \in \mathbb{Z}\}$

Divided by 4.

Handle the "4x." I solved for 4x & then divided everything by 4.

970-290-0550

$4x \neq \frac{3\pi}{2} + 2n\pi$

$x \neq \frac{3\pi}{8} + \frac{n\pi}{2}$ gets it.

Find the slope of the tangent to $f(x) = \sqrt{5x-1}$ @ $(1, 2)$

by approximating with the slope of a secant line.

Book would have you do $x_2 = 1$, then $x_1 = 0.5, 0.75, 0.9, 0.99, 0.999$

$x_1 = 1.5, 1.25, 1.1, 1.01, 1.001$

Ouch! Can't we do calculus, yet?

No!

$$\frac{f(x+h) - f(x)}{h} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$x_1 = 1$

Not 1, idiot!

$$\sqrt{5x-1} = f(x)$$

$$\Rightarrow f(x_2) = f(1) = \sqrt{5x-1} = \sqrt{5-1} = \sqrt{4} = 2$$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{2 - \sqrt{5x_2 - 1}}{1 - x_2} = \frac{f(x_1) - f(x_2)}{x_1 - x_2}$$

```
Y1(1.5)
           1.099019514
Y1(1.25
            1.16515139
Y1(1.1
           1.213203436
           1.213203436
Y1(1.01
           1.246117975
Y1(1.001
           1.249609619
Y1(1.0001
            1.24996094
```

```
           1.246117975
Y1(1.001
           1.249609619
Y1(1.0001
            1.24996094
Y1(1.000000001
                  1.25
```

$$f := x \to \frac{(2 - \text{sqrt}(5 \cdot x - 1))}{1 - x}$$

$$f := x \mapsto \frac{2 - \sqrt{5 \cdot x - 1}}{1 - x}$$

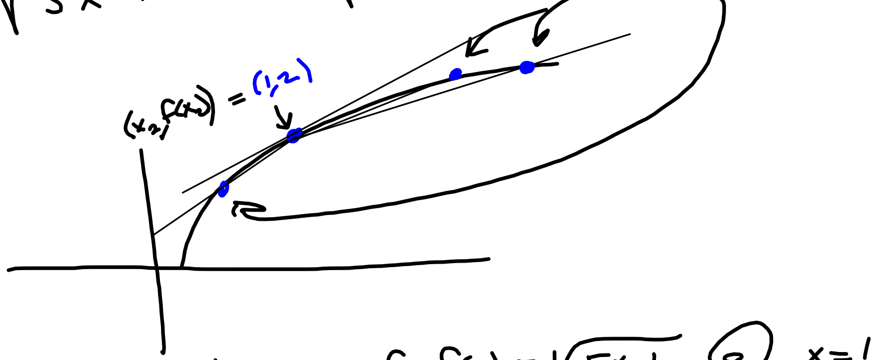
$[f(1.5), f(1.25), f(1.1), f(1.01), f(1.001), f(1.00001)]$

[1.099019514, 1.165151388, 1.213203440, 1.246118000, 1.249610000,
1.250000000]

$[f(0.5), f(.75), f(.9), f(.99), f(.999), f(.9999)]$

[1.550510258, 1.366750420, 1.291713070, 1.253930900, 1.250391000,
1.250040000]

$$\sqrt{5x-1} = \sqrt{5\left(x-\tfrac{1}{5}\right)}$$



Find the exact slope of $f(x) = \sqrt{5x-1}$ @ $x=1$, using a limit.

$$\frac{f(x_2)-f(x_1)}{x_2-x_1} = \frac{2-\sqrt{5x_1-1}}{1-x_1}$$

Rationalize the NUMERATOR.

want $x_1 \longrightarrow 1$

But $x_1 = 1$ is bad. Division by zero!

$(a-b)\cdot \quad (a+b) \qquad a^2-b^2$

$$\left(\frac{2-\sqrt{5x-1}}{1-x}\right)\left(\frac{2+\sqrt{5x-1}}{2+\sqrt{5x-1}}\right) = \frac{4-(5x-1)}{(1-x)(2+\sqrt{5x-1})}$$

$$= \frac{4-5x+1}{\sim\sim} = \frac{5-5x}{\sim\sim} = \frac{5\boxed{(1-x)}}{\boxed{(1-x)}(2+\sqrt{5x-1})} = \frac{5}{2+\sqrt{5x-1}}$$

$\xrightarrow{x\to 1}$ with impunity!

$$\frac{5}{2+\sqrt{5(1)-1}} = \frac{5}{2+\sqrt{4}} = \frac{5}{2+\sqrt{4}} = \frac{5}{2+2} = \boxed{\frac{5}{4} = m}$$

= exactly the slope of $\sqrt{5x-1}$ @ $x=1$.

Find slope of $f(x) = \frac{3}{x+5}$ @ any x

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{3}{x+h+5} - \frac{3}{x+5}}{h}$$

$$= \frac{1}{h}\left[\left(\frac{3}{x+h+5}\right)\left(\frac{x+5}{x+5}\right) - \left(\frac{3}{x+5}\right)\left(\frac{x+h+5}{x+h+5}\right)\right]$$

$$= \frac{1}{h}\left[\frac{3(x+5) - 3(x+h+5)}{(x+5)(x+h+5)}\right] = \frac{1}{h}\left[\frac{3x+15-3x-3h-15}{\sim\sim}\right]$$

$$= \frac{1}{h}\left[\frac{-3h}{\sim\sim\sim}\right] = \frac{-3}{(x+5)(x+\underbrace{h}_{0}+5)} \xrightarrow{h\to 0} \frac{-3}{(x+5)^2}$$

$= f'(x) =$ slope of $f(x)$ @ any x.

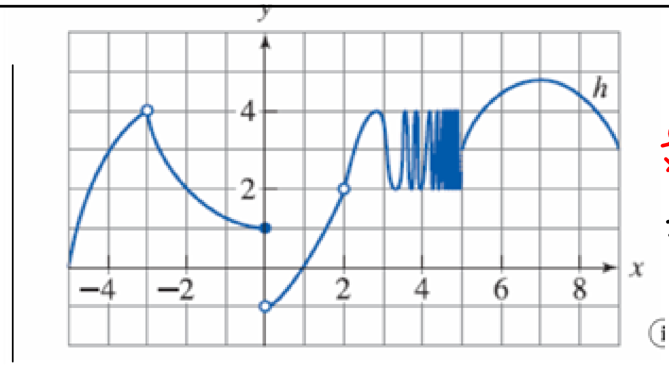
I just did $\lim\limits_{h\to 0} \frac{f(x+h) - f(x)}{h}$ = "f prime of x." $= f'(x)$.

Alternatively:

$$f'(c) = \lim_{x\to c} \frac{f(x) - f(c)}{x - c} = \lim_{x_2\to c} \frac{f(x_2) - f(c)}{x_2 - c}$$

$$f'(x) = \lim_{x_2\to x} \frac{f(x_2) - f(x)}{x_2 - x}$$

Don't waste too much time doing 1.4's analytically, although it'll build some muscles.



$\lim_{x \to -3} f(x) = 4$

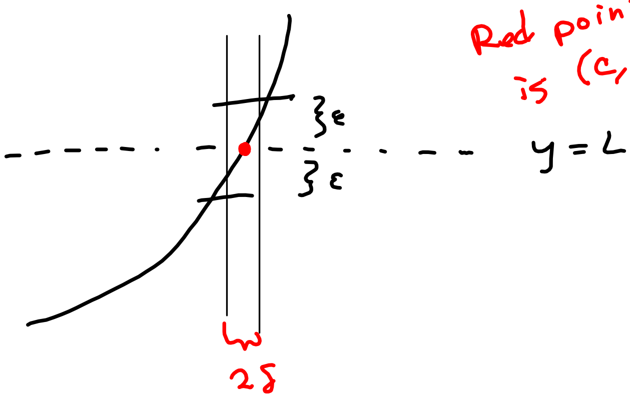
$\lim_{x \to -3^-} f(x) = 4 = \lim_{x \to -3^+} f(x)$

$\lim_{x \to 0^-} f(x) = 1 \neq -1 = \lim_{x \to 0^+} f(x)$

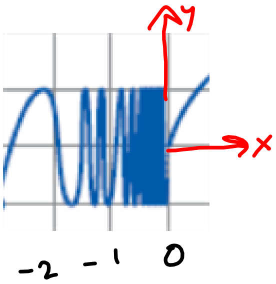
$\therefore \lim_{x \to 0} f(x) \nexists$

$\lim_{x \to c} f(x) = L$ means I can make $f(x)$ as close to L as I want, by taking x sufficiently close to c.

FORMALLY: means for any $\varepsilon > 0$, there is a $\delta > 0$ such that $0 < |x - c| < \delta$ implies $|f(x) - L| < \varepsilon$



Red point is (c, L)



has features of "topologist's sine curve."

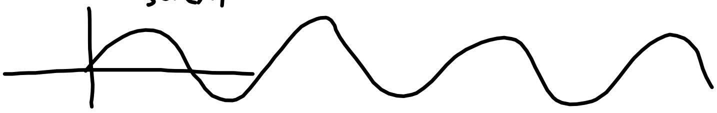
$\lim_{x \to 0^-} f(x) \nexists$ is the point, here.

Based on something like

$$\sin\left(\frac{\pi}{x}\right)$$

$$x \to 0^-, \quad \frac{\pi}{x} \to -\infty$$

$\sin(x)$



$\sin\left(\frac{\pi}{x}\right)$