§1.3 #11 Building sine/cosine from High-Low or Midline values

Cosine. They want sine. $a\cos(b(t-c))+d$

Period: $T=365$

want sin/cosine have period 2π

$bt = 2\pi$ when $t=365$

$b = \frac{2\pi}{t} = \frac{2\pi}{365}$

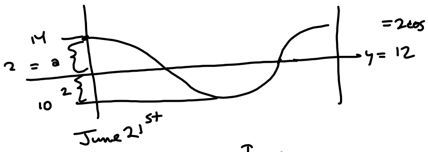
midline

$a\cos\left(\frac{2\pi}{365}(t-c)\right)+12$

How tall?

$= 2\cos\left(\frac{\pi}{365}(t-c)\right)+12$

$= 2\cos\left(\frac{\pi}{365}(t-172)\right)+12$



Jan Feb Mar Apr My June
31 28 31 30 31 21
$\underbrace{}_{151}$

172 days to get to 6/21

1.2 using this ↗ as an example.

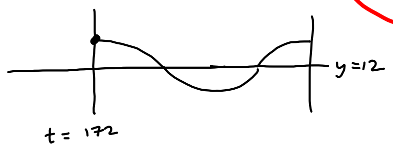
$2\cos\left(\frac{2\pi}{365}(t-172)\right)+12$

$\cos(t) = f(t)$ $g(t) = 2f\left(\frac{2\pi}{365}(t-172)\right)+12$

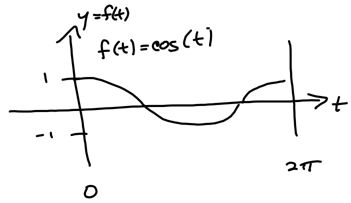
start

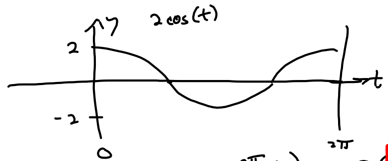
midline

multiply period by $\frac{365}{2\pi}$



$g(t) = 2f\left(\frac{2\pi}{365}(t-172)\right)+12$



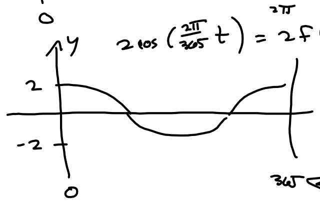


argument

$2\cos\left(\frac{2\pi}{365}t\right) = 2f\left(\boxed{\frac{2\pi}{365}t}\right)$

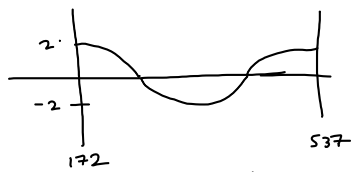
$\frac{365}{2\pi}\cdot t$ in graph

$\frac{365}{2\pi}\cdot 2\pi = 365$



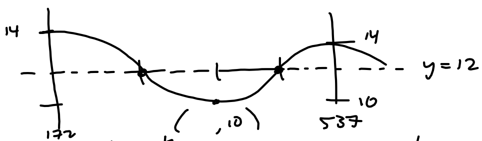
$2\cos\left(\frac{2\pi}{365}(t-172)\right)$ $t \mapsto t+172$

$365+172 = 537$



$2\cos\left(\frac{2\pi}{365}(t-172)\right)+12$ up 12

$y \mapsto y+12$



We start on the Summer Solstice for the high point.

$f(x)$ Basic function

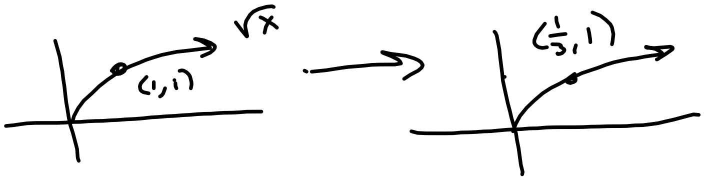
$af(x)$ $\quad y \longmapsto ay$

$f(x)=x^2 \qquad 2f(x)=2x^2$

$x^2 \longrightarrow 2x^2$

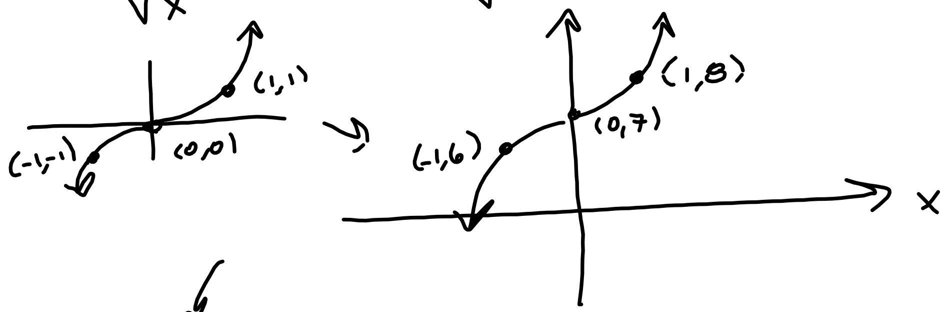
$f(bx)$ $\quad x \longmapsto \frac{1}{b}x$

$\sqrt{x} \longrightarrow \sqrt{3x} = \sqrt{3}\sqrt{x}$



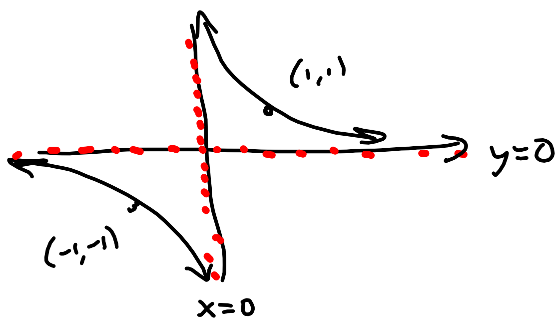
$f(x)+d$ $\quad y \longmapsto y+d$

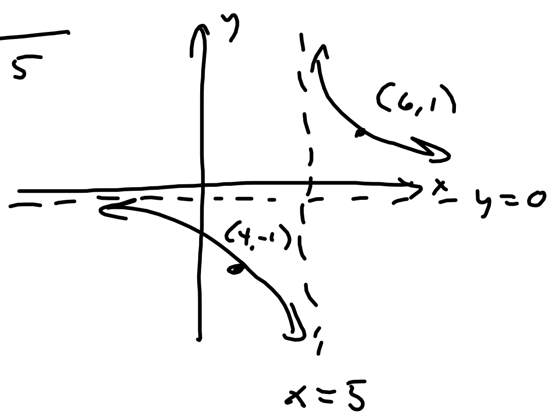
$\sqrt[3]{x} \longrightarrow \sqrt[3]{x}+7$ up 7



$f(x-c)$ $\quad x \longmapsto x+c$ (Delayed)

$f(x)=\frac{1}{x} \longrightarrow \frac{1}{x-5}$



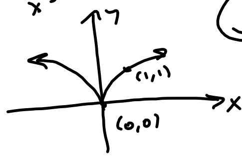


$g(x) = 3(2x+4)^{2/3} - 1 = 3(2(x+2))^{2/3} - 1$

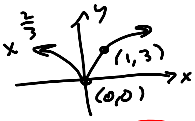
ORDER OF STEPS Matters:

M2	M1
$af(x)$	$af(x)$
$af(bx)$	$af(x-c)$
$af(b(x-c))$	$af(bx-c)$
$af(b(x-c))+d$	$af(bx-c)+d$
My Preference	Students Like.

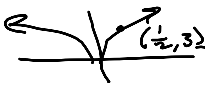
① $x^{\frac{2}{3}}$



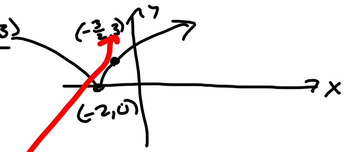
② $3x^{\frac{2}{3}}$



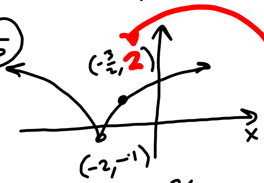
③ $3(2x)^{2/3}$



④ $3(2(x+2))^{2/3}$



⑤

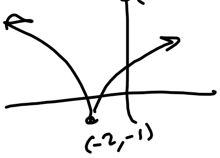


$3(2(x+3))^{2/3} - 1$

For the quick & dirty:

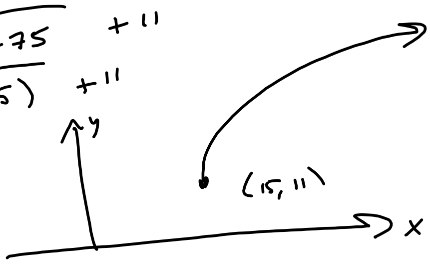
$3(2x+4)^{2/3} - 1 = 3(2(x+2))^{2/3} - 1$

Left 2 Down 1

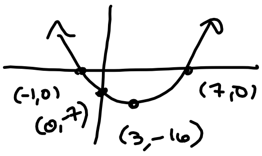


$7\sqrt{5x-75} + 11$

$7\sqrt{5(x-15)} + 11$



$x^2 - 6x - 7 = (x-7)(x+1)$



$(3-7)(3+1)$

$-4(4) = -16$

$f(x) = x^2 - 6x - 7$

$= \underline{x^2 - 6x + 3^2} - 9 - 7$

$= (x-3)^2 - 16 \rightsquigarrow (h,k) = (3,-16)$

$a(x-h)^2 + k$

$(h,k) = \text{vertex}$

$x^2 + bx + c$

$= \underline{x^2 + bx + (\frac{b}{2})^2} - \frac{b^2}{4} + c$

$(x + \frac{b}{2})^2 - \ldots$

$$\int \frac{dx}{\sqrt{x^2 - 6x - 7}} = \int \frac{dx}{\sqrt{(x-3)^2 - 16}}$$

Sums, Products, quotients & COMPOSITIONS

$\mathcal{D}$ = Domain.

$\mathcal{D}(f \pm g)$, $\mathcal{D}(fg)$, $\mathcal{D}(\frac{f}{g})$, $\mathcal{D}(f \circ g)$

$$\mathcal{D}(f \pm g) = \mathcal{D}(fg) = \mathcal{D}(f) \cap \mathcal{D}(g)$$

$$= \{x \mid x \in \mathcal{D}(f) \text{ AND } x \in \mathcal{D}(g)\}$$

$f(x) = \frac{1}{x^2-4}$ & $g(x) = \sqrt{x-1}$

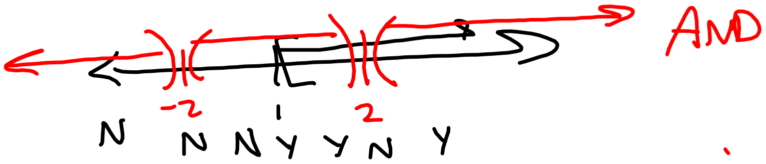
Need $x-1 \geq 0$

$x \geq 1$

Need $x^2-4 \neq 0$

$x \neq \pm 2$

$\mathbb{R} \setminus \{\pm 2\}$



$$\mathcal{D}(f+g) = \mathcal{D}\left(\frac{1}{x^2-4} + \sqrt{x-1}\right)$$

$= [1,2) \cup (2,\infty)$ Same for $f-g$, fg

$fg = (fg)(x) = f(x)\,g(x)$

$$\frac{f}{g} = \frac{\frac{1}{x^2-4}}{\sqrt{x-1}} = \frac{\frac{1}{x^2-4}}{\frac{\sqrt{x-1}}{1}} = \frac{1}{x^2-4} \cdot \frac{1}{\sqrt{x-1}}$$

$= \frac{1}{(x^2-4)\sqrt{x-1}}$ Same as $\mathcal{D}(f+g)$ EXCEPT $g(x) \neq 0$.

So $\mathcal{D}(g)$ AND $\underline{g(x) \neq 0 \text{ excludes}}$ $x=1$

So $\mathcal{D}(\frac{f}{g}) = (1,2) \cup (2,\infty)$ ← $x=1$ is Not ok

$\mathcal{D}(f+g) = [1,2) \cup (2,\infty)$ ↑ $x=1$ is OK

Next time:

$\mathcal{D}(f \circ g)$

$\mathscr{D}(f \circ g) = \mathscr{D}\left((f \circ g)(x)\right) = f(g(x))$ g is inside f!

NOT $f(x)g(x) = fg$

$(f \circ g)(x) = f(g(x)) = f(\square)$

$= \dfrac{1}{g(x)^2 - 4} = \dfrac{1}{\square^2 - 4} = \dfrac{1}{(\sqrt{x-1})^2 - 4}$

$= \dfrac{1}{x-1-4} = \dfrac{1}{x-5}$

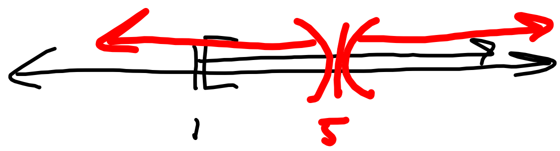
$\mathscr{D}(f \circ g) = \{ x \mid x \in \mathscr{D}(g) \text{ and } g(x) \in \mathscr{D}(f) \}$

$= \{ x \mid x \geq 1 \text{ and } \sqrt{x-1} \neq \pm 2 \}$

$\sqrt{x-1} \neq 2$ $\quad$ $\sqrt{x-1} \neq -2$ Never!

$x - 1 \neq 4$

$x \neq 5$



AND

$= [1,5) \cup (5, \infty) = \mathscr{D}(f \circ g)$