§ 1.1, 1.2 Reviewish

§1.1 Functions Represented 4 ways

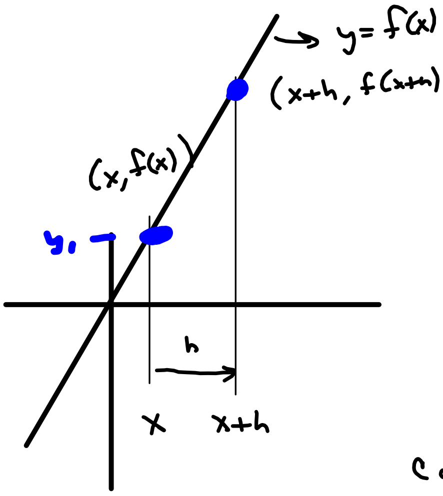
Verbally, Numerically, Graphically, $\boxed{\text{Algebraically}}$

sin(x) is not algebraic.

Formulaically

No formula? No Calculus

Recall, $f(x) = 2x$ has slope $m=2$



$$m = \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

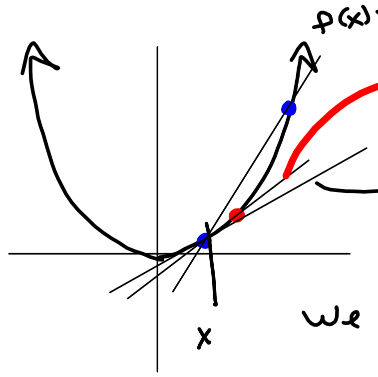
$$= \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{2(x+h) - 2x}{h}$$

$$= \frac{2x + 2h - 2x}{h} = \frac{2h}{h} = \boxed{2 = m}$$

Differential Calculus Generalizes this notion to smooth curves, using limits.

$f(x) = 2x \Rightarrow f(x+h) = 2(x+h)$

Consider $f(x) = x^2$



Secant Slope is an estimate

Tangent (Exact) slope @ x!
(Our Goal)

We want the Exact Slope @ x!

Bringing 2^{nd} Point closer gives better estimate
Making 2^{nd} the same as 1^{st} point gives $\frac{0}{0}$.

$$\cancel{x+h = x} \Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{f(x) - f(x)}{0} = \frac{0}{0} \text{ NO HELP.}$$

$h = 0$

$h = 0$ BAD
$h \to 0$ w/o touching Good.

Slope of $f(x) = x^2$: $(a+b)^2 = a^2 + 2ab + b^2$

DIFFERENCE QUOTIENT

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$$\text{SLOPE} = \frac{2xh + h^2}{h} = \frac{h(2x + h)}{h} = 2x + h \xrightarrow{h \to 0} 2x$$

It's not h = 0. It's h -> 0, which says "infinitely close without touching."

I can make 2x + h as close to 2x as I wish by making h sufficiently close to 0, *without touching*.

We can do this with the difference quotient of ANY SMOOTH FUNCTION!!!!!

Newton says so!

DOMAIN is ALL REAL #s, EXCEPT...

(1) Division by zero : $\frac{\text{STUFF}}{0}$ is Bad

METHOD : GIVEN $\frac{\text{STUFF}}{\text{JUNK}}$, we need Junk $\neq 0$.

(2) Negative values under an even-index radical

$\sqrt{\text{Negative}}$ is Bad. $\sqrt[2n]{\text{negative}}$

$\sqrt[4]{}$ $\sqrt[6]{}$ $\sqrt[8]{}$

METHOD $\sqrt{\text{JUNK}} \Longrightarrow$ Need Junk ≥ 0

What's the domain?

(1) $f(x) = \frac{9x^2-7}{x^2+3x+2}$ Need $x^2+3x+2 \neq 0 \Rightarrow$

$(x+2)(x+1) \neq 0 \Rightarrow$ DeMorgan's Laws for Sets & Logic

$x \neq (-1 \text{ OR } -2)$

$\underline{x \neq -1}$ AND $\underline{x \neq -2}$

Domain = $\mathcal{D}$ = $\{x \mid x \neq -1, x \neq -2\}$

webAssign

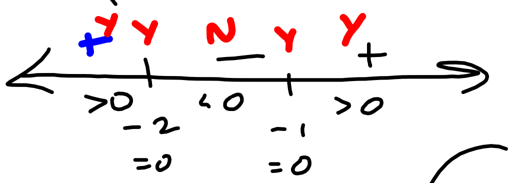
$= (-\infty,-2) \cup (-2,-1) \cup (-1,\infty)$

$= \mathbb{R} \setminus \{-1,-2\}$ I like

(2) $g(x) = \sqrt{x^2+3x+2}$

Need $x^2+3x+2 \ge 0$

$(x+2)(x+1) \ge 0$



Want ≥ 0

"OR"

$\mathcal{D} = (-\infty,-2] \cup [-1,\infty)$

$= \{x \mid x \le -2 \text{ OR } x \ge -1\}$

$h(x) = \dfrac{27\sin x + 53}{\sqrt{x^2+3x+2}}$

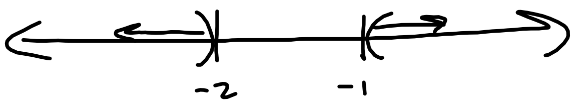
Need $x^2+3x+2 \ge 0$ AND $\sqrt{x^2+3x+2} \ne 0$ $\Big\}$ $x^2+3x+2 > 0$

$\boxed{\mathcal{D}(h) = (-\infty,-2) \cup (-1,\infty)}$

OR

$= \{x \mid x < -2 \text{ OR } x > -1\}$

Describe: $\{x \mid x < -2 \text{ AND } x > -1\}$

$=$ 

AND

$= \{\} = \emptyset =$ null set!

Stewart's Calculus 9th Edition is the book on which the WebAssign is based.

ANY edition will work and almost any calculus book will work.