Here's a much easier way to work this than the one I gave in class, today, 11/21/19:

$$\int \sec^5(x)\sin(x)\,dx = \int \cos^{-5}(x)\sin(x)\,dx =$$

$$u = \cos(x) \Rightarrow du = -\sin(x)\,dx \Rightarrow$$

$$-\int \left(\cos(x)\right)^{-5}\left(-\sin(x)\,dx\right) = -\int u^{-5}\,du = -\frac{1}{-4}u^{-4} + C =$$

$$\frac{1}{4}\cos^{-4}(x) + C = \frac{1}{4}\sec^4(x) + C$$

You could safely leave it in terms of cosine, so the very last step, converting to secant, wasn't necessary, unless there were an artificial condition, such as "Use only positive exponents in your final answer," which would be a pretty kooky thing to require for this one.