201 TEST #4 FALL, 2019

1 (10 pts) $\int_1^4 7x^2\,dx$

$\frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$, $x_k = a + k\Delta x = 1 + k\left(\frac{3}{n}\right) = 1 + \frac{3k}{n}$

$= \frac{3k+n}{n}$

$f(x_k) = 7x_k^2 = 7\left(\frac{3k+n}{n}\right)^2 = 7\left(\frac{9k^2+6kn+n^2}{n^2}\right)$

$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} 7x_k^2 = 7\Delta x \sum_{k=1}^{n} x_k^2$

$= 7\left(\frac{3}{n}\right)\sum_{k=1}^{n} \frac{9k^2+6kn+n^2}{n^2} = \frac{21}{n^3}\sum_{k=1}^{n}\left(9k^2+6kn+n^2\right)$

$= \frac{21}{n^3}\left[9\sum_{k=1}^{n} k^2 + 6n\sum_{k=1}^{n} k + n^2\sum_{k=1}^{n} 1\right]$

$= \frac{21}{n^3}\left[9\left(\frac{n^3+\cdots}{3}\right) + 6n\left(\frac{n^2+\cdots}{2}\right) + n^2(n)\right]$

$= \frac{21}{n^3}\left[3(n^3+\cdots) + 3n^3+\cdots + n^3\right]$

$= \frac{63n^3+\cdots}{n^3} + \frac{63n^3+\cdots}{n^3} + \frac{21n^3}{n^3}$

$\xrightarrow{n\to\infty} 126+21 = 147$

$\int_1^4 7x\,dx$

$= \frac{7}{3}\left[x^3\right]_1^4 = \frac{7}{3}(63)$

$= \frac{7}{3}\left[\frac{64-1}{1}\right]$

$= 147$ ✓

(2) 10 pts

$$\int_0^3 (9-x^2)\,dx = \left[9x - \tfrac{1}{3}x^3\right]_0^3$$

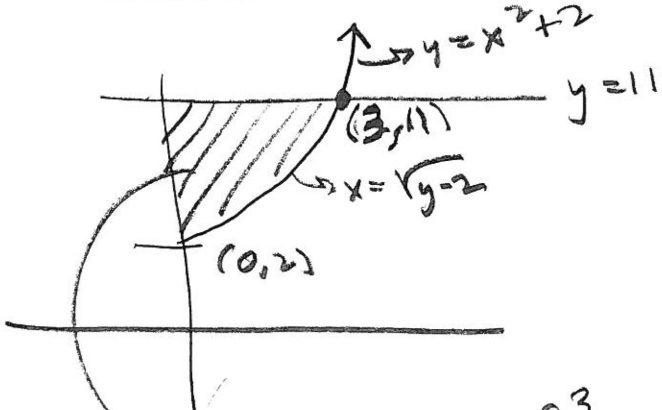
$$= 9(3) - \tfrac{1}{3}(3)^3 - 0 = 27 - 9 = 18$$

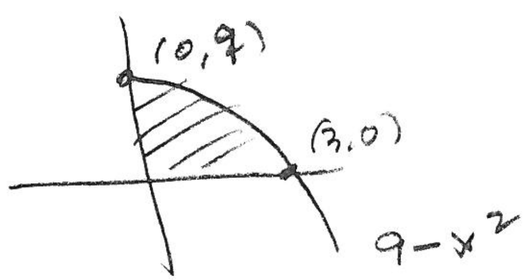
$$\int_2^{11} \sqrt{y-2}\,dy = \left.\frac{2(y-2)^{\frac{3}{2}}}{3}\right]_2^{11}$$

$$= \frac{2(11-2)^{\frac{3}{2}}}{3} - \frac{2(2-2)^{\frac{3}{2}}}{3} = \frac{2(9)^{\frac{3}{2}}}{3} = \frac{2(3)^3}{3} = 2(9) = 18$$

Bonus 4 5 pts

$\int_0^3 9-x^2$

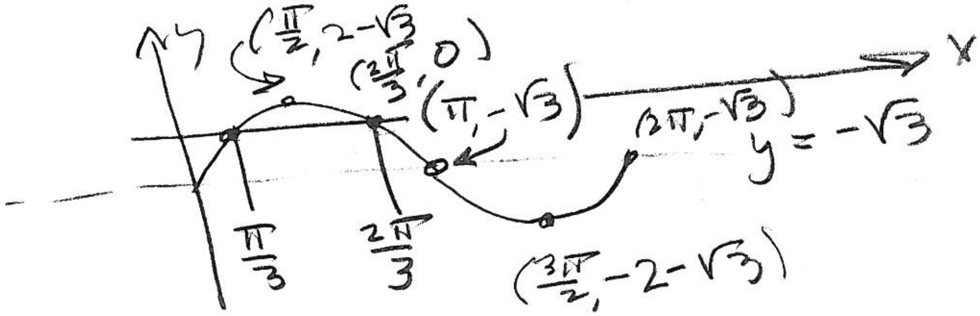




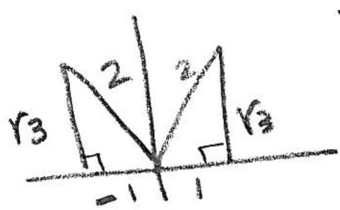
This area is $\int_0^3 (11 - (x^2+2))\,dx$

201 $\sqrt{Y}$ 3

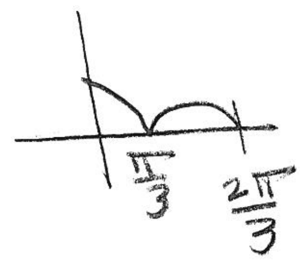
(3) 20 pts $$\int_0^{\frac{2\pi}{3}} \left| 2\sin(x) - \sqrt{3} \right| dx$$



$$2\sin x - \sqrt{3} = 0$$
$$2\sin x = \sqrt{3}$$
$$\sin x = \frac{\sqrt{3}}{2}$$



$$x = 60^\circ, 120^\circ$$
$$= \frac{\pi}{3}, \frac{2\pi}{3}$$



$$\int_0^{\frac{2\pi}{3}} = -\int_0^{\frac{\pi}{3}} + \int_{\frac{\pi}{3}}^{\frac{3\pi}{3}}$$

$$= -\int_0^{\frac{\pi}{3}} (2\sin x - \sqrt{3})\, dx + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} (2\sin x - \sqrt{3})\, dx$$

$$= -\left[-2\cos x - \sqrt{3}x \right]_0^{\frac{\pi}{3}} + \left[-2\cos x - \sqrt{3}x \right]_{\frac{\pi}{3}}^{\frac{2\pi}{3}}$$

$$= 2\cos\frac{\pi}{3} + \sqrt{3}\cdot\frac{\pi}{3} - \left(2\cos 0 - \sqrt{3}\cdot 0\right) + -2\cos\frac{2\pi}{3} - \sqrt{3}\cdot\frac{2\pi}{3}$$

$$- \left(-2\cos\frac{\pi}{3} - \sqrt{3}\left(\frac{\pi}{3}\right)\right) = 2\left(\frac{1}{2}\right) - \frac{\sqrt{3}\pi}{3} - 2 - 2\left(-\frac{1}{2}\right)$$

$$- \frac{2\pi\sqrt{3}}{3} + 2\cdot\frac{1}{2} + \frac{\sqrt{3}\pi}{3} = 1!$$

201 TY

(4) $\int_0^4 (3x-1)^5 dx = \frac{1}{3}\int_0^4 (3x-1)^5 (3\,dx)$

10pts

$u = 3x-1$
$du = 3dx$

$= \frac{1}{3}\left[\frac{(3x-1)^6}{6}\right]_0^4 = \frac{1}{18}\left[[3(4)-1]^6 - [3(0)-1]^6\right]$

$= \frac{1}{18}\left[11^6 - (-1)^6\right] = \frac{1}{18}\left[1,771,561 - 1\right]$

$= \frac{1,771,560}{18} = \boxed{98420}$

$\int_{\frac{1}{3}}^4 \approx 98420.05\overline{5}$.

$\int_0^{\frac{1}{3}} (3x-1)^5 dx = -.0555\overline{55}$

(5) (2) 20pts $\int x^2(3x-1)^5 dx = \frac{1}{3}\int \left(\frac{u+1}{3}\right)^2 (u^5\,du)$

$u = 3x-1$ $\quad 3x-1 = u$
$du = 3dx$ $\quad 3x = u+1$
$\quad x = \frac{u+1}{3}$
$\frac{dx}{3} = du$

$= \frac{1}{3}\left(\frac{u^2+2u+1}{9}\right)u^5\,du = \frac{1}{27}\int (u^7 + 2u^6 + u^5)\,du$

$= \frac{1}{27}\left[\frac{u^8}{8} + \frac{2}{7}u^7 + \frac{1}{6}u^6\right] + C$

$= \frac{1}{27}\left[\frac{(3x-1)^8}{8} + \frac{2(3x-1)^7}{7} + \frac{(3x-1)^6}{6}\right] + C$

$= \frac{(3x-1)^8}{216} + \frac{2(3x-1)^7}{189} + \frac{(3x-1)^6}{162} + C$

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5 b 20pts

$$\int \frac{6x-5}{\sqrt[3]{9x^2-15x}}\,dx = \frac{1}{3}\int (9x^2-15x)^{-\frac{1}{3}}\left(3(6x-5)\right)dx$$

$$= \frac{1}{3}\int u^{-\frac{1}{3}}\,du$$

$u = 9x^2 - 15x$

$du = (18x - 15)\,dx$

$= 3(6x-5)\,dx$

Need factor of 3

$$= \frac{1}{3}\cdot\frac{(9x^2-15x)^{\frac{2}{3}}}{\frac{2}{3}} + c = \frac{3}{2}\cdot\frac{1}{3}(9x^2-15x)^{\frac{2}{3}} + c$$

Variations:

$$= \frac{1}{2}(9x^2-15x)^{\frac{2}{3}} + c = \frac{1}{2}\left((9x^2-15x)^2\right)^{\frac{1}{3}} + c$$

$$= \frac{1}{2}\sqrt[3]{(9x^2-15x)^2} + c = \frac{1}{2}\left(\sqrt[3]{9x^2-15x}\right)^2 + c$$

201 T4

(6) (10pts) $\displaystyle\int_0^{\frac{\pi}{3}} \sec^5(x)\sin(x)\,dx = I$

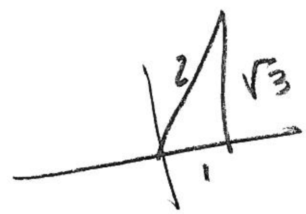
$\sec^5 x = \sec^4 x \sec x = \sec^4 x \dfrac{1}{\cos x}$

$\Longrightarrow$

$\sec^5 x \sin x\, dx = \sec^4 x \cdot \dfrac{1}{\cos x} \cdot \sin x$

$= \sec^4 x \tan x = (\sec^3 x)(\sec x \tan x)$

$= u^3\, du$ where $u = \sec x$.



So, $I = \displaystyle\int u^3\, du = \frac{1}{4}u^4 + C$

$= \boxed{\dfrac{1}{4}\sec^4 x + C}$ So $\dfrac{1}{4}\left[\sec^4 x\right]_0^{\frac{\pi}{3}}$

$\dfrac{1}{4}\left[\sec^4\left(\frac{\pi}{3}\right) - \sec^4(0)\right]$

$= \dfrac{1}{4}\left[2^4 - 1^4\right]$

$= \dfrac{1}{4}\left[16 - 1\right] = \boxed{\dfrac{15}{4}}$

(7) (a) (10pts) $\displaystyle\frac{d}{dx}\int_3^x \sin^3(t^2\cos(t))\,dt$

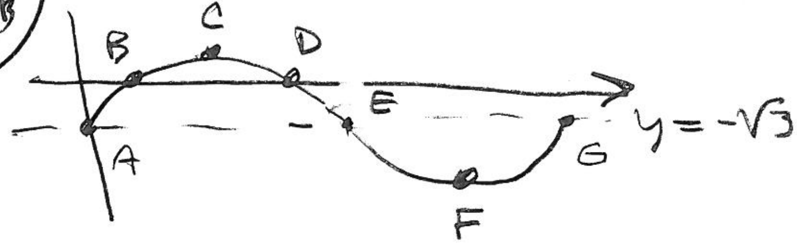
$= \boxed{\sin^3(x^2\cos x) + C}$

(7b) (10pts) $\displaystyle\frac{d}{dx}\int_0^{x^2+2x} \sin^3(t^2\cos(t))\,dt$

$= \boxed{\sin^3\left((x^2+2x)^2\cos(x^2+2x)\right)(2x+2)}$

201 JY

B1 10 pts $f(x) = 2\sin x - \sqrt{3}$



See Pg 3

$A = (0, -\sqrt{3}) \approx (0, -1.732)$ $\quad 2 - 1.732$

$B = (\frac{\pi}{3}, 0)$

$C = (\frac{\pi}{2}, 2-\sqrt{3}) \approx (3.142, .268)$ MAX

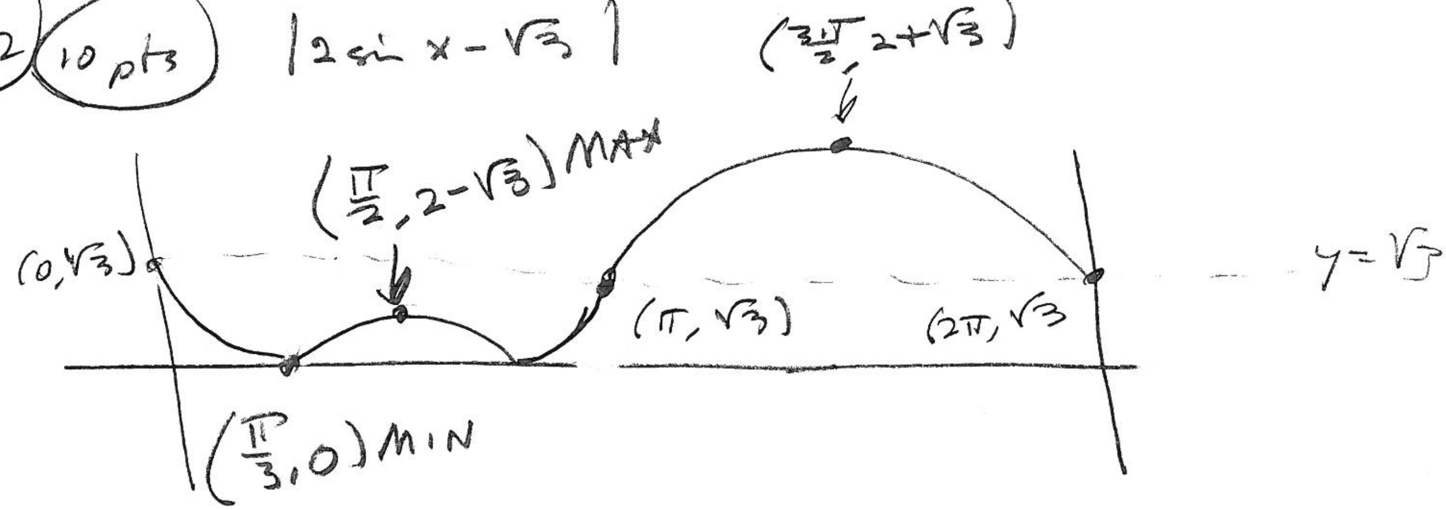
$D = (\frac{2\pi}{3}, 0)$

$E = (\pi, -\sqrt{3})$ IP

$F = (\frac{3\pi}{2}, -2-\sqrt{3}) \approx (4.712, -3.732)$ MIN

$G = (2\pi, -\sqrt{3}) \approx (2\pi, -1.732)$

B2 10 pts $|2\sin x - \sqrt{3}|$



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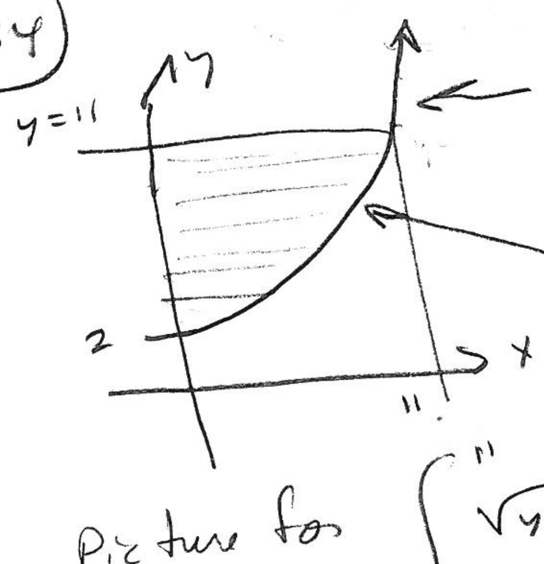
B3 5pts $\frac{d}{dx}\int_{x^5}^{x^2+2x} \sin^3(t^2\cos(t)) \, dt$

$$= \frac{d}{dx}\left[\int_{x^5}^{0} \sim + \int_{0}^{x^2+2x} \sim\right] = \frac{d}{dx}\left[-\int_{0}^{x^5}\sim + \int_{0}^{x^2+2x}\sim\right]$$

$$= \boxed{-\sin^3\left(\left(x^5\right)^2\cos\left(x^5\right)\right)\left(5x^4\right) + \left(\sin^3\left(\left(x^2+2x\right)^2\cos\left(x^2+2x\right)\right)\right)(2x+2)}$$

B4



$x = \sqrt{y-2}$

$x^2 = y-2$

$x^2+2 = y$

Picture for $\int_2^{11} \sqrt{y-2} \, dy$

Integral for Picture as $f(x)$-situation:

$$\int_0^3 \left(11 - \left(x^2+2\right)\right) dx = \int_0^3 \left(9 - x^2\right) dx \,!$$

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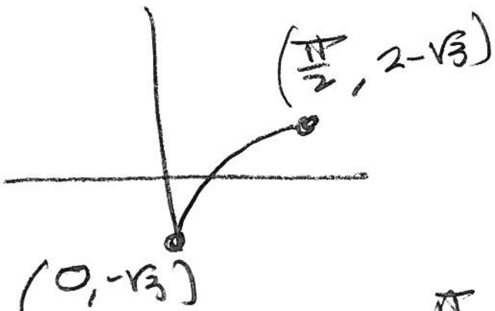
TY

B5

5 pts

$$\int_0^{\frac{\pi}{2}} (2\sin x - \sqrt{3})\,dx$$



$b - a = \frac{\pi}{2} - 0 = \frac{\pi}{2}$

$$-\sqrt{3}\left(\frac{\pi}{2}\right) \le \int_0^{\frac{\pi}{2}} f(x)\,dx \le (2-\sqrt{3})\left(\frac{\pi}{2}\right)$$

B6

5 pts

$f(x) = 2\sin x - \sqrt{3}$ is a trig function & cnts & difbl on its domain

Its domain is $\mathbb{R}$ $\Longrightarrow$

cnts on $[0, \frac{\pi}{2}]$ &

difbl on $(0, \frac{\pi}{2})$ $\Longrightarrow$ MVT applies

$f(0) = -\sqrt{3}$, $f(\frac{\pi}{2}) = 2 - \sqrt{3}$ $\Longrightarrow$

$$\frac{f(b) - f(a)}{b - a} = \frac{2 - \sqrt{3} - (-\sqrt{3})}{\frac{\pi}{2} - 0} = \frac{2}{\pi}(2) = \frac{4}{\pi}$$

$f'(x) = 2\cos x \overset{\text{SET}}{=} \frac{2}{\pi}$

$\cos x = \frac{1}{\pi}$ $\longrightarrow$

$$\boxed{c = x = \arccos\left(\frac{1}{\pi}\right) \approx 1.24685022 \approx c}$$

201 JY

B7 5pts $f(x) = \sqrt{5x} \Longrightarrow$

$$\frac{f(x+h)-f(x)}{h} = \frac{\sqrt{5(x+h)} - \sqrt{5x}}{h}$$

$$= \frac{\sqrt{5(x+h)} - \sqrt{5x}}{h} \cdot \frac{\sqrt{5(x+h)} + \sqrt{5x}}{\sqrt{5(x+h)} + \sqrt{5x}}$$

$$= \frac{5x + 5h - 5x}{h\left(\sqrt{5(x+h)} + \sqrt{5x}\right)} = \frac{5\cancel{h}}{\cancel{h}\left(\sqrt{5(x+h)} + \sqrt{5x}\right)}$$

$$\xrightarrow{h \to 0} \frac{5}{\sqrt{5x} + \sqrt{5x}} = \boxed{\frac{5}{2\sqrt{5x}} = f'(x)}$$

B8 10pts $\cos(32^\circ) = \cos\left((30^\circ)\left(\frac{\pi}{180^\circ}\right) + \frac{2\pi}{180^\circ}\right)$

$f(x) = \cos x \Longrightarrow f'(x) = -\sin x$

Let $x_1 = 30^\circ = \frac{\pi}{6}$ $\quad f(x_1) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

$x_2 = \frac{\pi}{6} + \frac{\pi}{90}$ $\quad f'(x) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$$L_{x_1}(x) = f'(x_1)(x - x_1) + f(x_1)$$

$$= -\frac{1}{2}\left(x - \frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}$$

$$\Longrightarrow L(x_2) = \cos(32^0) \approx -\frac{1}{2}\left(\frac{\pi}{6} + \frac{\pi}{90} - \frac{\pi}{6}\right) + \frac{\sqrt{3}}{2}$$

$$= \boxed{-\frac{1}{2}\left(\frac{\pi}{90}\right) + \frac{\sqrt{3}}{2} \approx .8485721113}$$

Actual: .8480480962

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B9 SpB Look at any of the last couple months' tests & lecture notes.