201 Test 3 Take-Home FALL, 2019

① $f(x) = 16x^3 - 24x^2 - 63x + 98$

$$\begin{array}{r|rrrr} -2 & 16 & -24 & -63 & 98 \\ & & -32 & 112 & -98 \\ \hline & 16 & -56 & 49 & \end{array}$$

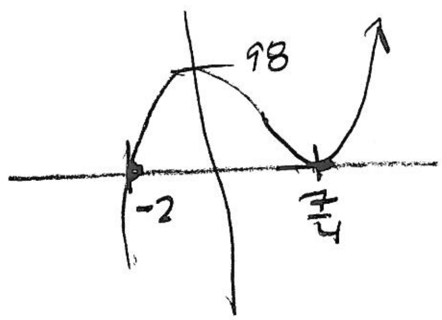
$x=-2$ by inspection of graphing app.

$16x^2 - 56x + 49$

$(4x)^2 - 2(4x)(7) + 7^2 = (4x-7)^2 \overset{SET}{=} 0 \longrightarrow \boxed{x = \frac{7}{4}}$

$2(4x)(7) = 56x$ ✓

$m=2$



OR $a=16, b=-56, c=49$

$b^2-4ac = (-56)^2 - 4(16)(49)$

$= 3136 - 3136 = 0$

$x = \frac{-(-56) \pm \sqrt{0}}{2(16)}$

$= \frac{7}{4}$, repeated root.

$f'(x) = 48x^2 - 48x - 63$

$$\begin{array}{l|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array} \qquad \begin{array}{l|l} 3 & 63 \\ 3 & 21 \\ & 7 \end{array}$$

$= 3(16x^2 - 16x - 21)$

$\overset{SET}{=} 0 \longrightarrow x^2 - x - \frac{21}{16} = 0 \quad \left(\frac{4+21}{16} = \frac{25}{16}\right)$

$x^2 - x + (\frac{1}{2})^2 - \frac{1}{4} \cdot \frac{4}{4} - \frac{21}{16} = 0$

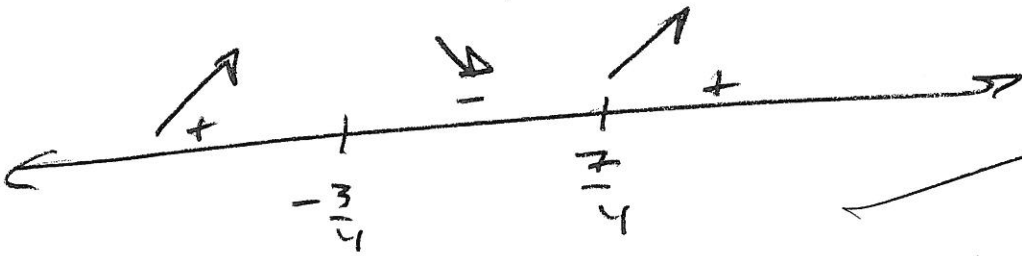
$(x-\frac{1}{2})^2 = \frac{25}{16}$

$x = \frac{1}{2} \pm \sqrt{\frac{25}{16}} = \frac{1}{2} \pm \frac{5}{4} = \frac{2 \pm 5}{4} \begin{array}{l} \nearrow \frac{7}{4} \\ \searrow -\frac{3}{4} \end{array}$

201 T3 Y H

1 cont'd $f'(x) = 48x^2 - 48x - 63 = 0$

2 $x = -\frac{3}{4}, \frac{7}{4}$



$$\boxed{\begin{array}{l} \text{MAX: } \left(-\frac{3}{4}, 125\right) \\ \text{MIN } \left(\frac{7}{4}, 0\right) \end{array}}$$

$-\frac{3}{4}$	16	−24	−63	98
		−12	27	27
	16	−36	−36	125

$\frac{7}{4}$	16	−24	−63	98
		28	7	−98
	16	4	−56	0

confirms work on f.

$\frac{(\cancel{56}\ 28\ 14)(7)}{\cancel{4}\ \cancel{2}\ 1} = 98$

$\frac{1}{2}$	16	−24	−63	98
		8	−8	$-\frac{71}{2}$
	16	−16	−71	$\frac{175}{2}$

$f''(x) = 96x - 48 \overset{\text{SET}}{=} 0$

$96x = 48$

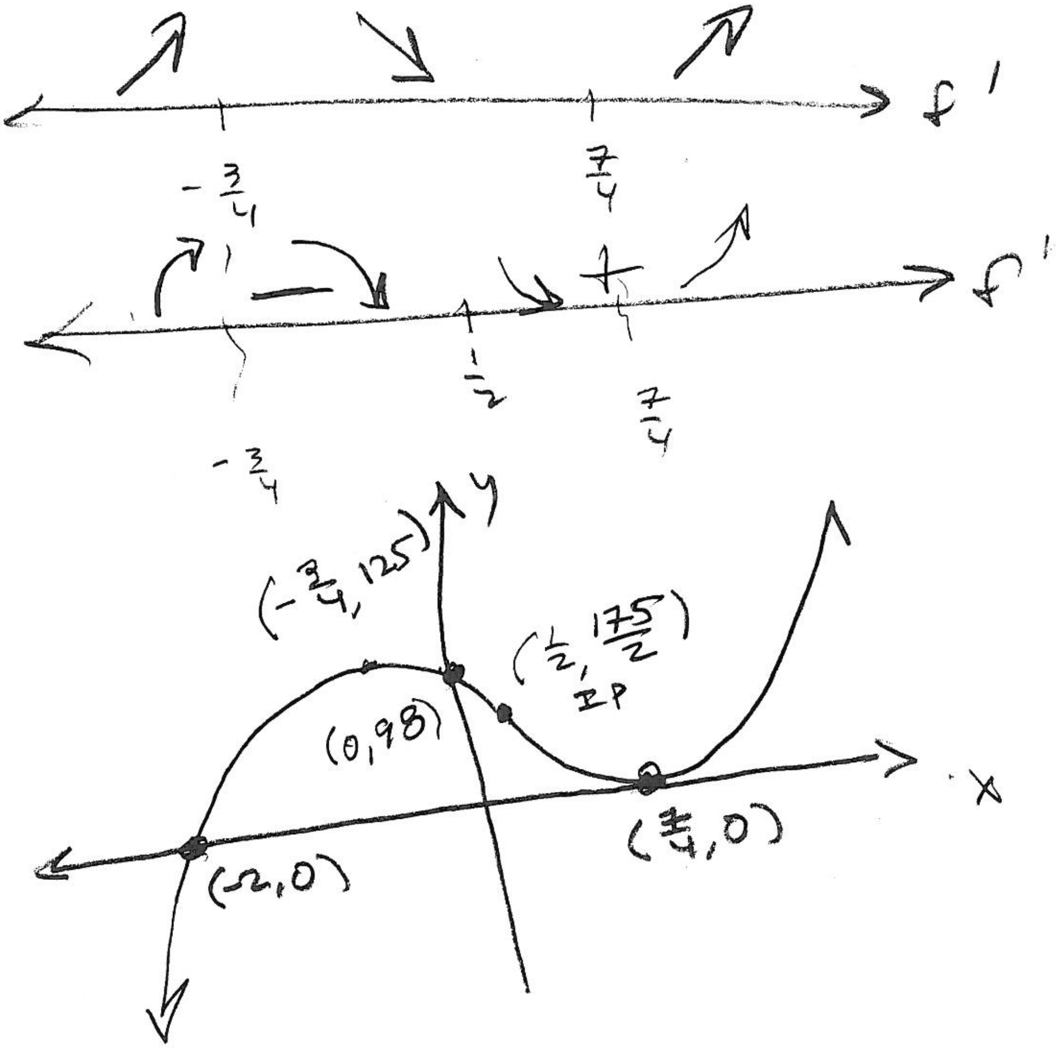
$x = \frac{48}{96} = \frac{1}{2} = x$

$98 - \frac{21}{2} = \frac{196 - 21}{2} = \frac{175}{2}$

$$\boxed{\begin{array}{c} \text{IP:} \\ \left(\frac{1}{2}, \frac{175}{2}\right) \end{array}}$$

$\frac{1}{2}$

1 cont'd



f'
$-\frac{3}{4}$
$\frac{7}{4}$
f''
$-\frac{3}{4}$
$\frac{1}{2}$
$\frac{7}{4}$
y
$(-\frac{3}{4}, 125)$
$(\frac{1}{2}, \frac{175}{2})$
IP
(0, 98)
x
(-2, 0)
$(\frac{7}{4}, 0)$

201 T3TH

2 10 pts $R(x) = \frac{x^2 - x - 6}{x+4} = \frac{(x+3)(x-2)}{x+4}$

$\mathcal{D} = \mathbb{R} \setminus \{-4\}$ $\boxed{\text{V.A } x = -4}$

$\boxed{\text{x-int: } (-3,0), (2,0)}$

y-int: $\frac{-6}{4} = -\frac{3}{2} \rightarrow \boxed{(0, -\frac{3}{2}) = \text{y-int}}$

O.A.:

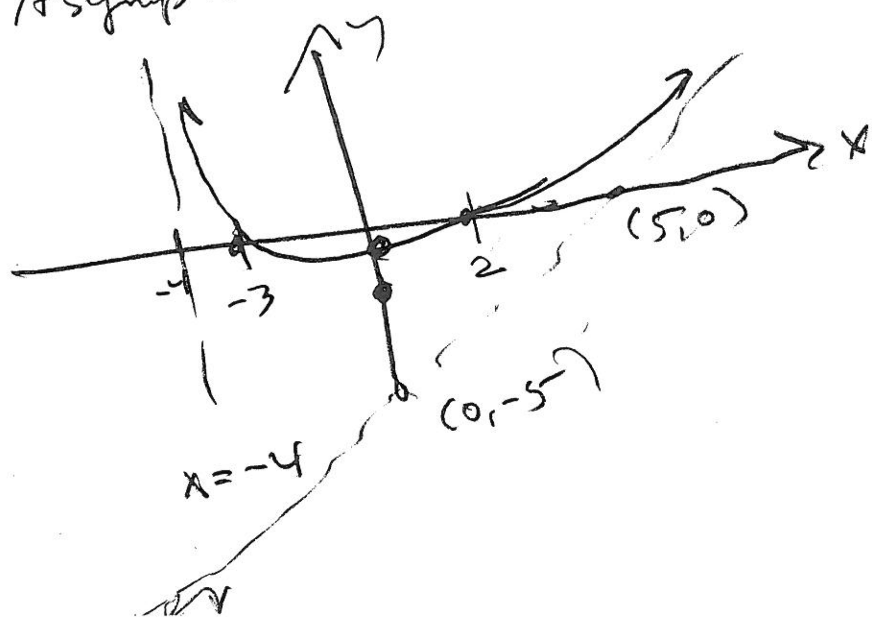
$$\begin{array}{r|rrr} -4 & 1 & -1 & -6 \\ & & -4 & 20 \\ \hline & 1 & -5 & 14 \end{array}$$

$\rightarrow y = x - 5$ is oblique Asymptote.

Sign Pattern for f(x):

$$\begin{array}{ccccccc} - & & + & & - & & + \\ & -4 & & -3 & & 2 & \end{array}$$

Asymptotes



.201 T3TH

$$R'(x) = \frac{(2x-1)(x+4) - (x^2 - x - 6)(1)}{(x+4)^2}$$

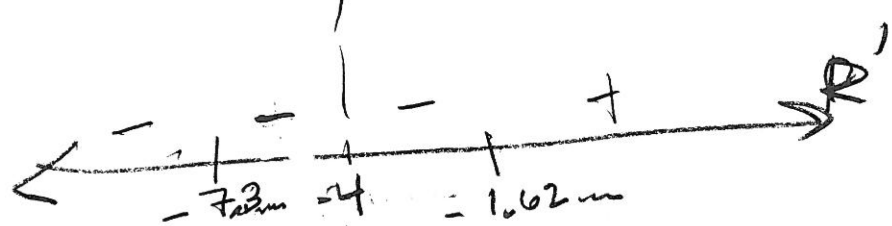
$$= \frac{2x^2+8x-x-4-x^2+x+6}{(x+4)^2} = \boxed{\frac{x^2+9x+2}{(x+4)^2} = R'(x)}$$

$$x^2+9x+12 = 0$$

$$x^2+9x+\left(\frac{9}{2}\right)^2 = -12+\frac{81}{4} = \frac{-48+81}{4} = \frac{33}{4}$$

$$\left(x+\frac{9}{2}\right)^2 = \frac{33}{4}$$

$$x = \frac{-9 \pm \sqrt{33}}{2}$$



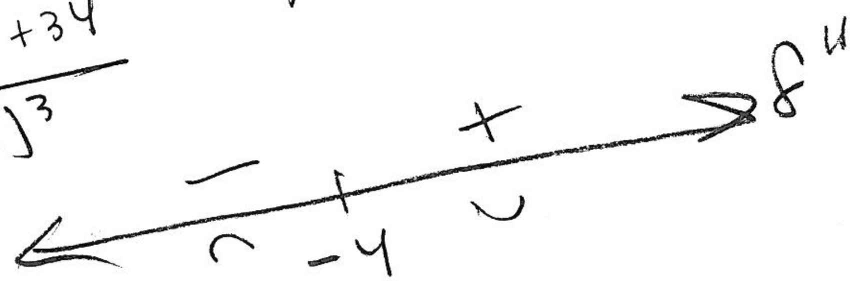
≈ -1.627718677,
-7.372281323

$$R''(x) = \frac{(2x+9)(x+4)^2 - ((x^2+9x+2)(2)(x+4)}{(x+4)^4}$$

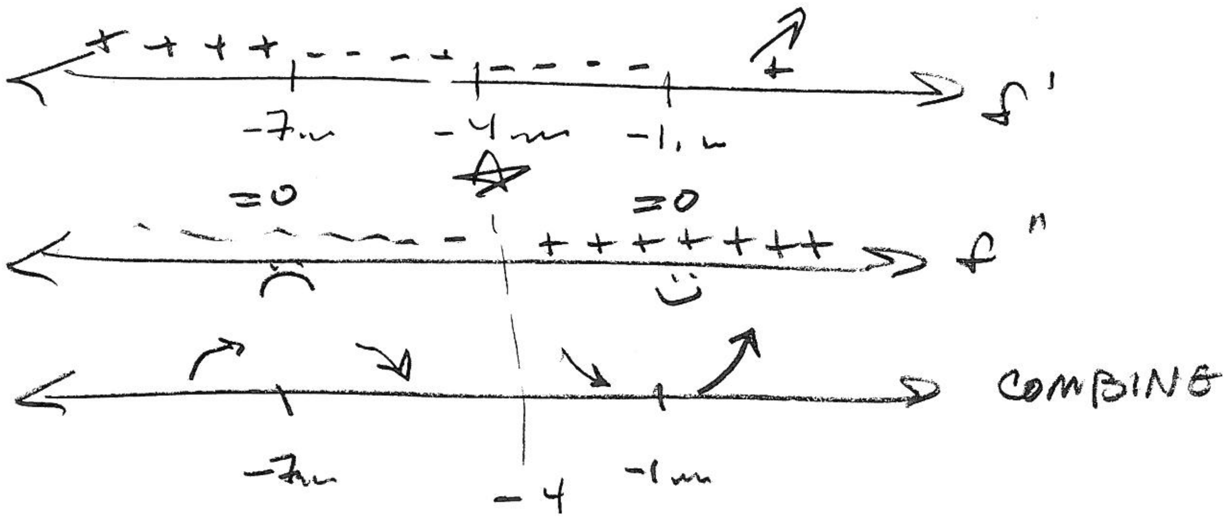
$$= \frac{(2x+9)(x+4) - x^2 - 9x - 2}{(x+4)^3} = \frac{2x^2+8x+9x+36-x^2-9x-2}{(x+4)^3}$$

$$x^2+8x+4^2 = -34+16 < 0$$

$$= \frac{x^2+8x+34}{(x+4)^3}$$



T3TH

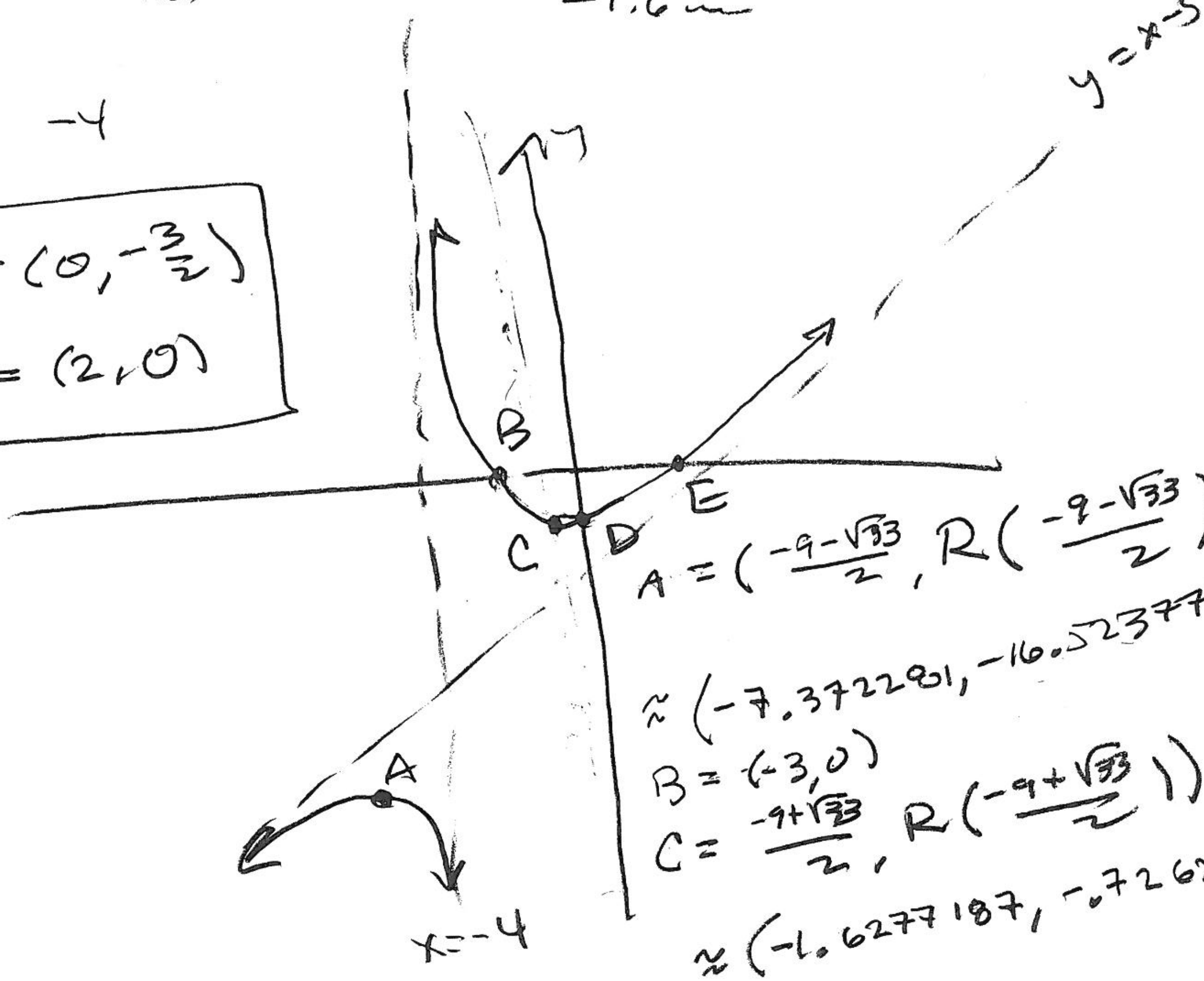


f', $\dfrac{-9-\sqrt{33}}{2}$, -4, $\dfrac{-9+\sqrt{33}}{2}$

-7.… -1.6…

f'' -4

$$\boxed{\begin{array}{l} D=\left(0,-\tfrac{3}{2}\right) \\ E=(2,0) \end{array}}$$



$A=\left(\dfrac{-9-\sqrt{33}}{2}, R\left(\dfrac{-9-\sqrt{33}}{2}\right)\right)$

$\approx(-7.372281, -16.523774)$

$B=(-3,0)$

$C=\dfrac{-9+\sqrt{33}}{2}, R\left(\dfrac{-9+\sqrt{33}}{2}\right)\Big)$

$\approx(-1.6277187, -.726226$

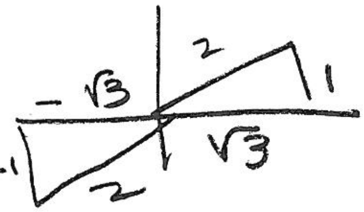
201 T3TH

(3) (10 pts) $g(x) = \sin x + \sqrt{3}\cos x + 1$

$g'(x) = \cos x - \sqrt{3}\sin x = 0 \longrightarrow$

$\cos x = \sqrt{3}\sin x \longrightarrow$

$\frac{\sin x}{\cos x} = \tan x = \frac{1}{\sqrt{3}}$

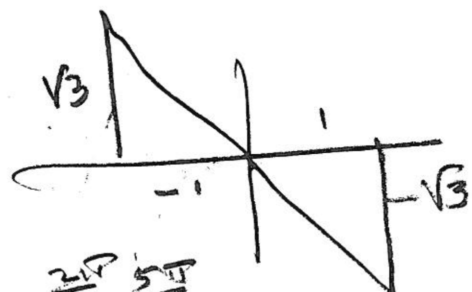


$\Rightarrow x = \frac{\pi}{6}, \frac{7\pi}{6}$ Note: $g'(0) = 1 > 0$ +

$\Rightarrow g''(x) = -\sin x - \sqrt{3}\cos x$

$= 0 \longrightarrow$

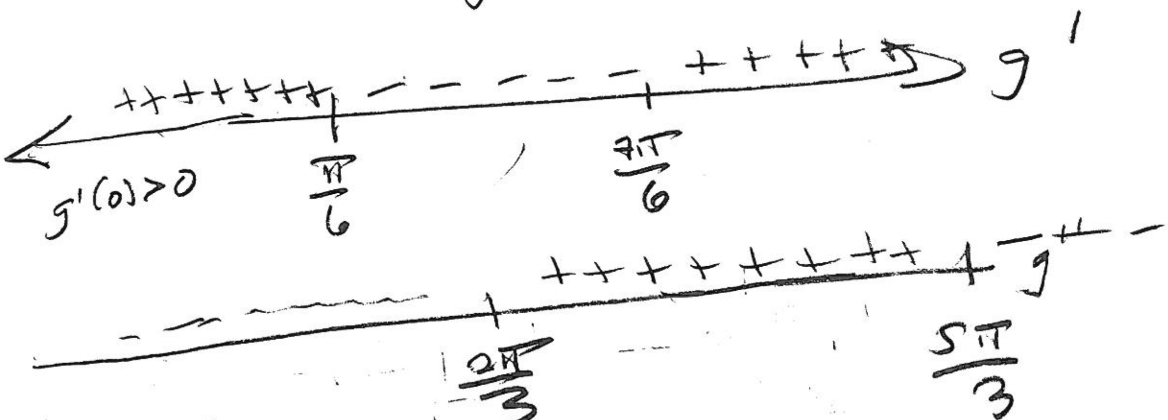
$-\sin x = \sqrt{3}\cos x$



$\Rightarrow -\tan x = \sqrt{3} \Rightarrow \tan x = -\sqrt{3}$

$\frac{2\pi}{3}, \frac{5\pi}{3}$

$\Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$ Note $g''(0) < 0$ −



COMBINE

201 $\sqrt{3}\,\pi$ H

③ cont'd

$0 = g(x_1) + g'(x_1)(x - x_1)$

$g'(x_1)x_2 - g'(x_1)x_1 + g(x_1) = 0$

$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)}$

Key pts

$f' = 0$: $x = \frac{\pi}{6}, \frac{7\pi}{6}$

$g\left(\frac{\pi}{6}\right) = \sin\frac{\pi}{6} + \sqrt{3}\left(\cos\frac{\pi}{6}\right) + 1 = \frac{1}{2} + \frac{\sqrt{3}\cdot\sqrt{3}}{2} + 1 = 3$

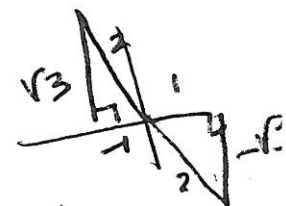
$g\left(\frac{7\pi}{6}\right) = \sin\frac{7\pi}{6} + \sqrt{3}\left(\cos\frac{7\pi}{6}\right) + 1 = -\frac{1}{2} - \frac{\sqrt{3}\sqrt{3}}{2} + 1 = -1$

$\left(\frac{\pi}{6}, 3\right)$ MAX

$\left(\frac{7\pi}{6}, 1\right)$ Min

$g\left(\frac{2\pi}{3}\right) = \sin\frac{2\pi}{3} + \sqrt{3}\cos\frac{2\pi}{3} + 1 = \frac{\sqrt{3}}{2} + \sqrt{3}\left(-\frac{1}{2}\right) + 1$

$= 1$



$g\left(\frac{5\pi}{3}\right) = \sin\frac{5\pi}{3} + \sqrt{3}\cos\frac{5\pi}{3} + 1 = -\frac{\sqrt{3}}{2} + \sqrt{3}\left(+\frac{1}{2}\right) + 1$

$= 1$

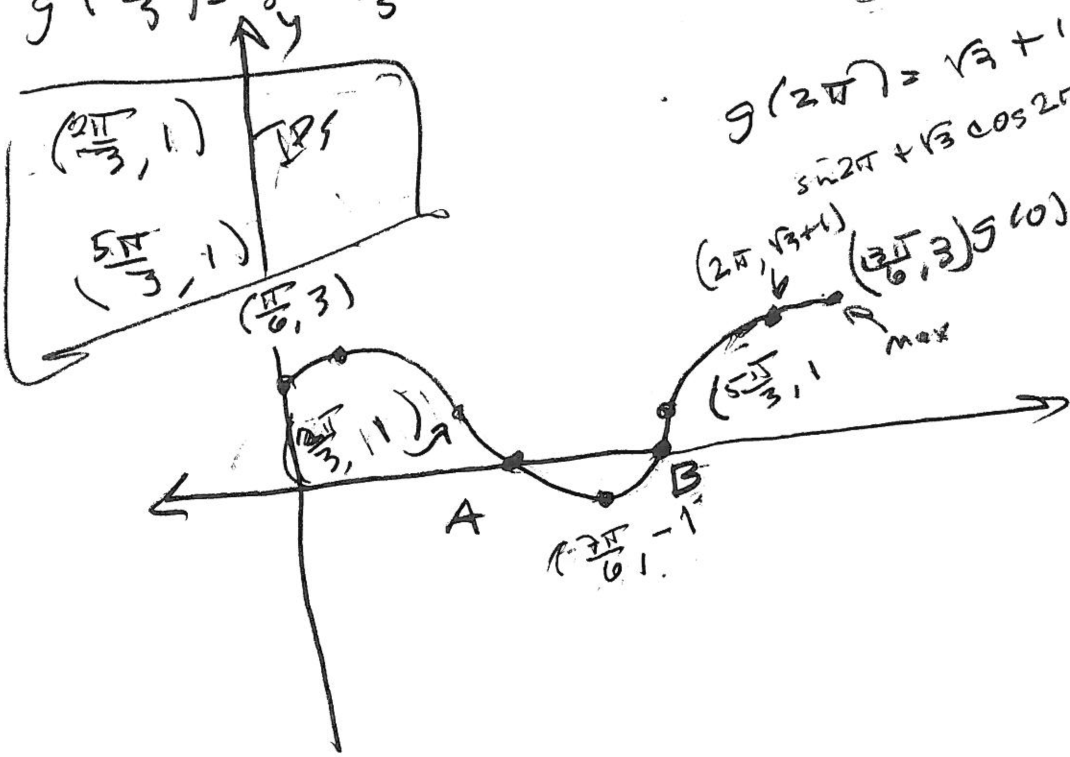
$g(2\pi) = \sqrt{3} + 1$

$\sin 2\pi + \sqrt{3}\cos 2\pi + 1 = 2$

$g(0) = \sqrt{3}\cos(0) + 1$

$= \sqrt{3} + 1$

≈ 2.732



3 2 201 2cn T3TH

3 cnt'd

Try the x-ints Hmmm $\cos(\frac{\pi}{2})=0$

$\sin\frac{\pi}{2}=1$

$\sin x+\sqrt{3}\cos x+1=0$? $\sin(\frac{3\pi}{2})=-1$!

$\sqrt{3}\cos x = 1-\sin x$ So $(\frac{3\pi}{2},0)$ &

$\sqrt{3} = \sec x-\tan x$ $\sin\frac{5\pi}{6}+\sqrt{3}\cos\frac{5\pi}{6}+1$

$\sin x = \cos(x-\frac{\pi}{2}) = \cos x\cos(-\frac{\pi}{2}) - \sin x\sin(-\frac{\pi}{2})$

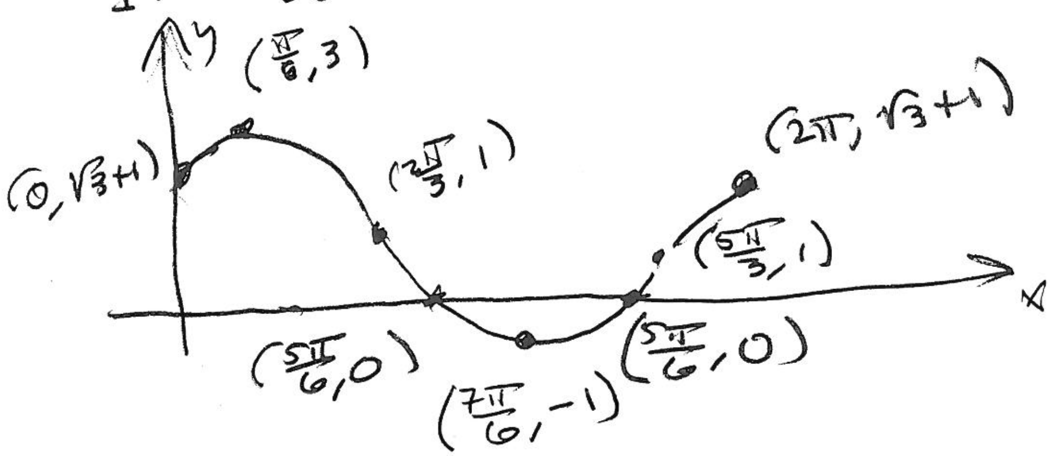
$= \frac{1}{2}-\sqrt{3}\cdot\frac{\sqrt{3}}{2}+1 = \frac{1}{2}-\frac{3}{2}+1=0$!

$= 0+\sin x$ $(\frac{5\pi}{6},0)$

OR Try Newton's w/ spreadsheet.

IP is $(\frac{2\pi}{3},1)$ for $x_1=\frac{2\pi}{3}$ guess (seed)

IP is $(\frac{5\pi}{3},1)$ for $x_1=\frac{5\pi}{3}$ seed



201 T3TH

(4) Bonus 5pts

$$T(x) = \frac{6x^3 - 29x^2 - 20x + 75}{x^2 + x - 12}$$

$$\begin{array}{r|rrrr} 5 & 6 & -29 & -20 & 75 \\ & & 30 & 5 & -75 \\ \hline & 6 & 1 & -15 & 0 \end{array}$$

$\frac{75}{12} = \frac{25}{4}$

$6x^2 + x - 15 = 6x^2 + 10x - 9x - 15$

$(3)(2) \quad -(3)(5) \quad = 2x(3x+5) - 3(3x+5)$

$= (3x+5)(2x-3)$

$x = 5, -\frac{5}{3}, \frac{3}{2}$

$$\frac{6(x-5)(x+\frac{5}{3})(x-\frac{3}{2})}{(x+4)(x-3)}$$

$\boxed{\begin{array}{l} x=-4 \quad V.A. \\ x=3 \end{array}}$

$$\begin{array}{r} 6x - 35 \\ x^2+x-12 \overline{\smash{)}\, 6x^3 - 29x^2 - 20x + 75} \\ -(6x^3 + 6x^2 - 72x) \\ \hline -35x^2 \end{array}$$

$\boxed{\begin{array}{c} y = 6x - 35 \\ O.A. \end{array}}$

$$T'(x) = \frac{3(2x^4 + 4x^3 - 75x^2 + 182x + 55)}{(x^2+x-12)^2}$$

$\overset{SET}{=} 0 \longrightarrow$

$x \approx -2.7145254 66$

OR -8.029031532

$$T''(x) = \frac{6(29x^3 - 345x^2 + 699x - 1147)}{(x^2+x-12)^3}$$

$(x+4)^3(x-3)^3$

$T''(x) = 0 \Longrightarrow$

$x \approx 9.858576088,$

201 T3TH

\# 4 cnt'd

A $\approx (-8.02903, -106.65781)$

B $= \left(-\frac{5}{3}, 0\right)$

C $\approx (-.27145, -6.41087)$

D $= \left(0, -\frac{25}{4}\right)$

E $= \left(\frac{3}{2}, 0\right)$

F $= (5, 0)$

G $\approx (9.859058, 29.54541)$

H $= \left(\frac{115}{29}, -\frac{325}{29}\right)$

$\approx (3.96552, -11.206897)$