MAT 201 Test 3, Fall, 2019 Name______________________
100 Points Covers Chapter 3

You know the drill. And remember to circle final answers.

1. Let $f(x)=x^3-9x^2+15x-135$ for the following problems:

   a. (25 pts) Sketch the graph of $f(x)$. Show all extreme points and inflection points. I *also* expect to see the *x*-intercept(s) and *y*-intercept. Work in the following order:
   1st derivative, critical point(s), sign pattern – 4 pts
   2nd derivative, inflection point(s), sign pattern – 4 pts
   General shape of *f* – 4 pts
   x-intercept(s), sign pattern – 3 pts
   y-intercept(s) – 1 pt
   General location of *f* at the extreme(s) and the inflection point(s) – 4 pts
   Precise value of *f* at the extremes and the inflection point(s) – 1 pt
   Put it all together in the graph, with proper labels (ordered-pair labels) of key points – 4 pts

   b. (10 pts) Find the maximum and minimum of $f(x)$ on the interval $[0,3]$.

   c. (10 pts) Confirm that the hypotheses of the Mean Value Theorem hold for $f(x)=x^3-9x^2+15x-135$ on $[0,3]$, and find the c that is promised in the conclusion of the theorem.

2. (10 pts) Find all local extremes of $g(x)=\cos(x)\sin(x)+\sin(x)$ in the interval $[0,2\pi)$.

3. (10 pts) Sketch the graph of a function g that has all the properties given:

$$\lim_{x\to -2^-} g(x)=\infty, \lim_{x\to -2^+} g(x)=-\infty$$
$$g(1)=0,$$
$$g'(2)=0, g(2)=3,$$
$$g'(x)>0 \ \forall x\in(-\infty,-2)\cup(2,\infty),$$
$$g'(x)<0 \ \forall x\in(-\infty,-2)\cup(2,\infty)$$
$$g''(3)=0, g(3)=2,$$
$$g''(x)>0 \ \forall x\in(-\infty,-2)\cup(3,\infty)$$
$$g''(x)<0 \ \forall x\in(3,\infty)$$

4. (5 pts) Evaluate $\lim_{x\to\infty}\left(\sqrt{25x^2-11x}-5x\right)$.

Bonus Answer up to 3 Bonus questions.

Bonus 1 (5 pts) Sketch the graph of $R(x)=\dfrac{x^2+2x-15}{x+2}$, showing all intercepts and asymptotes. This problem requires no calculus.

Bonus 2 (5 pts) Minimize the distance between $g(x)=5x-3$ and the point $(-8,1)$.

Bonus 3 (5 pts) Derive the recursion formula for Newton's method and use the figure, below to illustrate how x_2 is obtained from x_1.

Bonus 4 (5 pts) Use a differential to estimate the maximum error in the calculated volume of a sphere whose measueed radius is 10 cm, if the error in measurement could be as large as 0.1 cm.

Bonus 5 (5 pts) Use the tangent line to approximate $\sin(33^0)$. Remember to convert to radians! Degrees don't play nice with derivatives. Don't simplify.

Bonus 6 (5 pts) Find $\dfrac{dy}{dx}$ if $x^2-3xy+y^2-6=x^2y^3+8$. Then find an equation of the tangent line to the curve at $\left(1,-\dfrac{13}{3}\right)$.