MAT 201
100 Points

Test 3, Fall, 2019
Covers Chapter 3

Name KEY

You know the drill. And remember to circle final answers.

1. Let $f(x)=x^3-9x^2+15x-135$ for the following problems:

   a. (25 pts) Sketch the graph of $f(x)$. Show all extreme points and inflection points. I *also* expect to see the x-intercept(s) and y-intercept. Work in the following order:
   1st derivative, critical point(s), sign pattern – 4 pts
   2nd derivative, inflection point(s), sign pattern – 4 pts
   General shape of f – 4 pts
   x-intercept(s), sign pattern – 3 pts
   y-intercept(s) – 1 pt
   General location of f at the extreme(s) and the inflection point(s) – 4 pts
   Precise value of f at the extremes and the inflection point(s) – 1 pt
   Put it all together in the graph, with proper labels (ordered-pair labels) of key points – 4 pts

   b. (10 pts) Find the maximum and minimum of $f(x)$ on the interval $[0,3]$.

   c. (10 pts) Confirm that the hypotheses of the Mean Value Theorem hold for $f(x)=x^3-9x^2+15x-135$ on $[0,3]$, and find the c that is promised in the conclusion of the theorem.

2. (10 pts) Find all local extremes of $g(x)=\cos(x)\sin(x)+\sin(x)$ in the interval $[0,2\pi)$.

3. (10 pts) Sketch the graph of a function g that has all the properties given:

$$\lim_{x\to -2^-} g(x)=\infty,\ \lim_{x\to -2^+} g(x)=-\infty$$
$$g(1)=0,$$
$$g'(2)=0, g(2)=3,$$
$$g'(x)>0\ \forall x\in(-\infty,-2)\cup(2,\infty),$$
$$g'(x)<0\ \forall x\in(-\infty,-2)\cup(2,\infty),$$
$$g''(3)=0, g(3)=2,$$
$$g''(x)>0\ \forall x\in(-\infty,-2)\cup(3,\infty)$$
$$g''(x)<0\ \forall x\in(3,\infty)$$

4. (5 pts) Evaluate $\lim_{x\to\infty}\left(\sqrt{25x^2-11x}-5x\right)$.

Bonus Answer up to 3 Bonus questions.

Bonus 1 (5 pts) Sketch the graph of $R(x)=\dfrac{x^2+2x-15}{x+2}$, showing all intercepts and asymptotes. This problem requires no calculus.

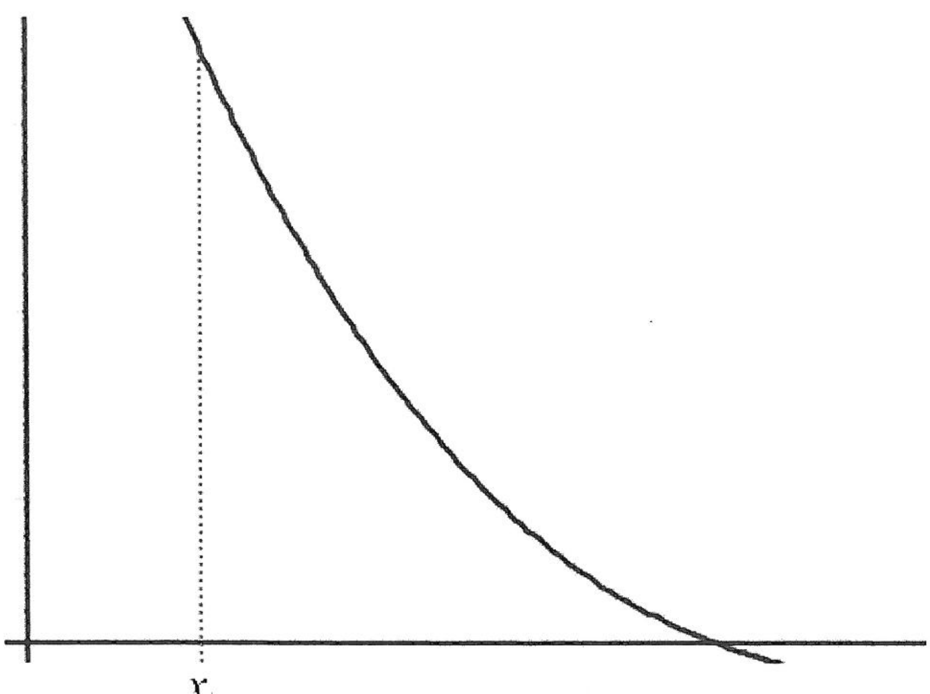
Bonus 2 (5 pts) Minimize the distance between $g(x)=5x-3$ and the point $(-8,1)$.

Bonus 3 (5 pts) Derive the recursion formula for Newton's method and use the figure, below to illustrate how x_2 is obtained from x_1.

Bonus 4 (5 pts) Use a differential to estimate the maximum error in the calculated volume of a sphere whose measueed radius is 10 cm, if the error in measurement could be as large as 0.1 cm.

Bonus 5 (5 pts) Use the tangent line to approximate $\sin\left(33^0\right)$. Remember to convert to radians! Degrees don't play nice with derivatives. Don't simplify.

Bonus 6 (5 pts) Find $\dfrac{dy}{dx}$ if $x^2-3xy+y^2-6=x^2y^3+8$. Then find an equation of the tangent line to the curve at $\left(1,-\dfrac{13}{3}\right)$.



201 TEST 3 FALL, 2019

① $f(x) = x^3 - 9x^2 + 15x - 135$

a 25pts

$\longrightarrow f'(x) = 3x^2 - 18x + 15 \overset{\text{set}}{=} 0 \longrightarrow$

$3(x^2 - 6x + 5) = 0 \longrightarrow$

$(x-5)(x-1) = 0 \longrightarrow$

$\boxed{x = 1, 5 \text{ critical}}$

$$\begin{array}{ccccccc} & + & & - & & + & \\ \longleftarrow & & | & & | & & \longrightarrow f' \\ & \nearrow & 1 & \searrow & 5 & \nearrow & \end{array}$$

$\longrightarrow f''(x) = 6x - 18 \overset{\text{set}}{=} 0 \longrightarrow$

$\boxed{x = 3 \text{ Inflection}}$

$$\begin{array}{ccccc} & - & & + & \\ \longleftarrow & & | & & \longrightarrow f'' \\ & \frown & 3 & \smile & \end{array}$$

$$\begin{array}{r|rrrr} 1 & 1 & -9 & 15 & -135 \\ & & 1 & -8 & 7 \\ \hline & 1 & -8 & 7 & -128 \end{array}$$

$(1, -128)$ MAX

$$\begin{array}{r|rrrr} 5 & 1 & -9 & 15 & -135 \\ & & 5 & -20 & -25 \\ \hline & 1 & -4 & -5 & -160 \end{array}$$

$(5, -160)$ MIN

$$\begin{array}{r|rrrr} 3 & 1 & -9 & 15 & -135 \\ & & 3 & -18 & -9 \\ \hline & 1 & -6 & -3 & -144 \end{array}$$

$(3, -144)$ INF

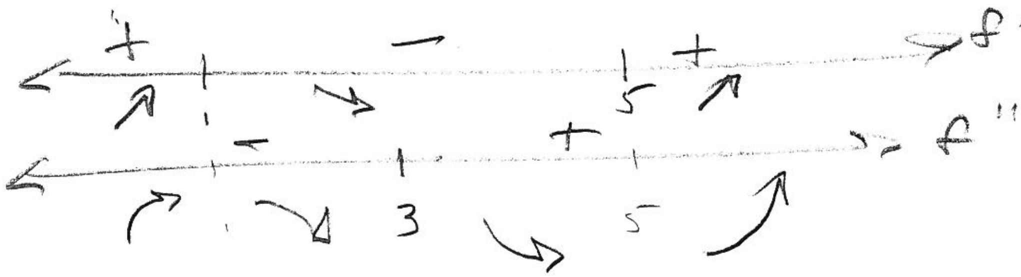
$f(x) = x^3 - 9x^2 + 15x - 135$

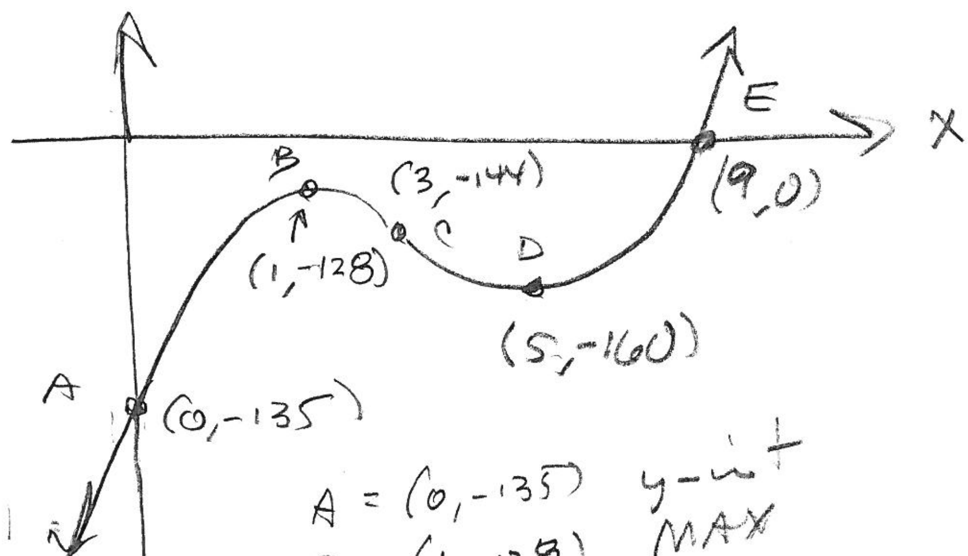
$= x^2(x-9) + 15(x-9)$

$= (x-9)(x^2+15) \overset{\text{SET}}{=} 0$

$x = 9 \rightsquigarrow \boxed{(9,0) \text{ x-int}}$

$f(0) = -135 \rightsquigarrow (0,-135)$ y-int





A = (0, -135) y-int

B = (1, -128) MAX

C = (3, -144) INFL

D = (5, -160) MIN

E = (9, 0) x-int

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1b 10 pts $f(0) = -135$

$$\boxed{\begin{array}{ll} f(3) = -144 & \text{MIN} \\ f(1) = -128) & \text{MAX} \end{array}}$$

Using previous work.

c 10 pts $f(x)$ is a polynomial $\Rightarrow$

f is cnts on $(-\infty, \infty) \supset [0,3]$

& f is difbl on $(-\infty, \infty) \supset (0,3)$

$\Rightarrow$ MVT hypotheses are satisfied on $[0,3]$.

So $\dfrac{f(3) - f(0)}{3-0} = \dfrac{-144-(-135)}{3} = \dfrac{-9}{3} = -3$

$f'(x) = 3x^2 - 18x + 15 \overset{\text{SET}}{=} -3 \Rightarrow$

$3x^2 - 18x + 18 = 0 \Rightarrow$

$3(x^2 - 6x + 6) = 0 \Rightarrow$

$b^2 - 4ac = (-6)^2 - 4(1)(6) = 36 - 24 = 12$

$\sqrt{12} = 2\sqrt{3} \Rightarrow$

$x = \dfrac{6 \pm 2\sqrt{3}}{2} = 3 \pm \sqrt{3} \Rightarrow$

$$\boxed{c = 3 - \sqrt{3}}$$

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T3

(4)

(2) $g(x) = \cos x \sin x + \sin x$ on $[0, 2\pi]$

$g(0) = 0$

$g(2\pi) = 0$

$g'(x) = -\sin^2 x + \cos^2 x + \cos x$

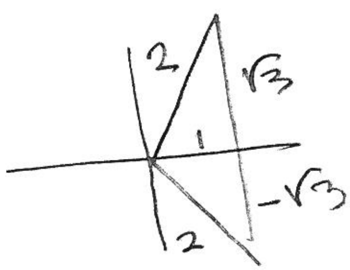
$= -(1 - \cos^2 x) + \cos^2 x + \cos x$

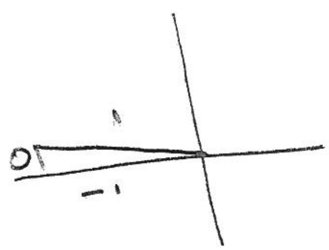
$= \cos^2 x - 1 + \cos^2 x + \cos x$

$= 2\cos^2 x + \cos x - 1 \overset{\text{SET}}{=} 0$

$\Longrightarrow (2\cos x - 1)(\cos x + 1) = 0$

$\Longrightarrow \cos x = \frac{1}{2}, \quad \cos x = -1$





$x = \frac{\pi}{3}, \frac{5\pi}{3}$ $\qquad$ $x = \pi$

$g\left(\frac{\pi}{3}\right) = \cos\frac{\pi}{3}\sin\frac{\pi}{3} + \sin\frac{\pi}{3}$

$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \frac{\sqrt{3}}{2} = \frac{\sqrt{3} + 2\sqrt{3}}{4} = \boxed{\frac{3\sqrt{3}}{4}}$ MAX

$g(\pi) = \cos\pi \sin\pi + \sin\pi = 0$

$g\left(\frac{5\pi}{3}\right) = \left(\frac{1}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right) = \frac{-\sqrt{3} - 2\sqrt{3}}{4}$

$= \boxed{\frac{-3\sqrt{3}}{4}}$ MIN

4 10 pts

$$\sqrt{25x^2-11x} - 5x = f(x)$$

$$= \left(\frac{\sqrt{25x^2-11x} - 5x}{1}\right)\left(\frac{\sqrt{25x^2-11x} + 5x}{\sqrt{25x^2-11x} + 5x}\right)$$

$$= \frac{25x^2-11x-25x^2}{\sqrt{25x^2-11x} + 5x} = \frac{-11x}{|5x|\sqrt{1-\frac{11x}{25x^2}} + 5x}$$

$$= \frac{-11x}{5x\left(\sqrt{1-\frac{11}{25x}} + 1\right)}$$

$$= \frac{-11}{5\left(\sqrt{1-\frac{11}{25x}} + 1\right)} \xrightarrow{x \to \infty} \frac{-11}{5(\sqrt{1}+1)}$$

$$= \frac{-11}{5(2)} = \boxed{\frac{-11}{10} = \lim_{x\to\infty} f(x)}$$

201 T3 (5)

B1 $R(x) = \dfrac{x^2+2x-15}{x+2} = \dfrac{(x+5)(x-3)}{x+2} \Rightarrow$

x-int: $(-5,0)$, $(3,0)$

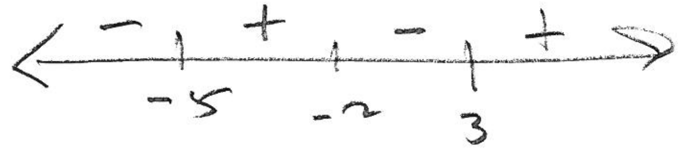
y-int: $(0,-\frac{15}{2})$

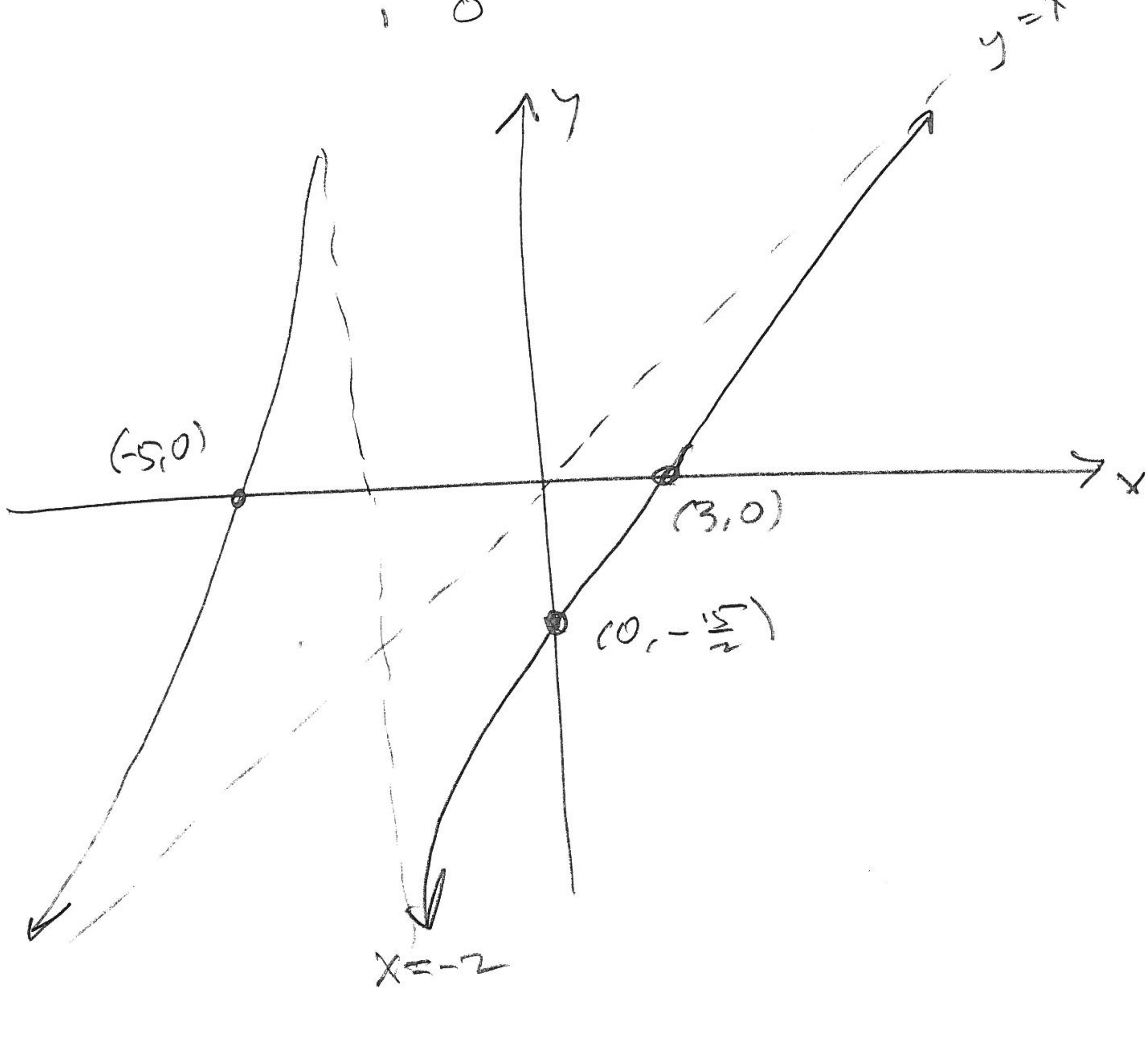
V.A.: $x=-2$

O.A.:

$$\begin{array}{r|rrr} -2 & 1 & 2 & -15 \\ & & -2 & \\ \hline & 1 & 0 & \end{array}$$

$\Rightarrow y = x$ is O.A.





201 J3 5

B2 $d = \sqrt{(x-(-8))^2 + ((5x-3)-1)^2}$

$(5x-4)^2$

we m~~a~~inimize $f(x) = d^2$

$= x^2 + 16x + 64 + 25x^2 - 40x + 16$

$= 26x^2 - 24x + 80 \Rightarrow$

$f'(x) = 52x - 24 \stackrel{SET}{=} 0 \Rightarrow$

$x = \frac{24}{52} = \frac{12}{26} = \frac{6}{13}$

$f(\frac{6}{13}) = \frac{32}{13} \Rightarrow d = \sqrt{\frac{32}{13}}$

$\begin{array}{r} {}^{2}13 \\ 8 \\ \hline 104 \end{array}$

$104 - 72 = 32$

$$\begin{array}{r|rrr} \frac{6}{13} & 26 & -24 & 80 \\ & & 12 & \frac{-72}{13} \\ \hline & 26 & -12 & \frac{32}{13} \end{array}$$

$\sqrt{\frac{32}{13}} = \frac{\sqrt{16 \cdot 2}}{\sqrt{13}} = \frac{4\sqrt{2}}{\sqrt{13}} = \frac{4\sqrt{2}\sqrt{13}}{13} = \boxed{\frac{4\sqrt{26}}{13} = d}$

Minimum

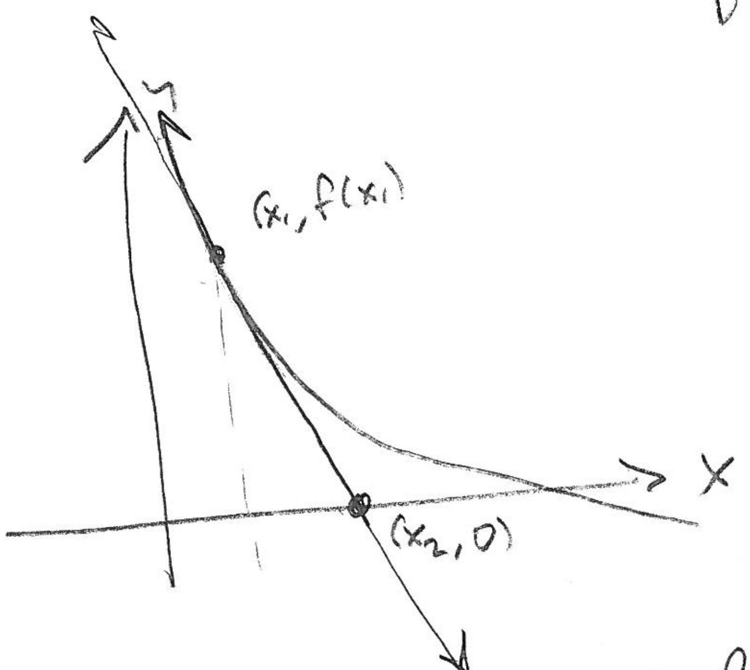
B3 x_2 is where tangent line to $f(x)$ at $x=x_1$ has its x-int:

Tangent line $= f'(x_1)(x-x_1) + f(x_1) \stackrel{\text{set}}{=} 0$

$$\Rightarrow f'(x_1)x - f'(x_1)x_1 + f(x_1) = 0$$

$$\Rightarrow f'(x_1)x = f'(x_1)x_1 - f(x_1)$$

$$\Rightarrow x_1 = \frac{f'(x_1)x_1 - f(x_1)}{f'(x_1)} = \boxed{x_1 - \frac{f(x_1)}{f'(x_1)} = x_2}$$



tangent line to $f(x)$ @ $x=x_1$

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T3

(6)

(34) $V = \frac{4}{3}\pi r^3$, $r = 10$, $\Delta r = 0.1 = dx$

$\frac{dV}{dr} = 4\pi r^2 \longrightarrow$

$\Delta V \approx dV = 4\pi r^2 dx = 4\pi (10)^2 (0.1)$

$= (400\pi)(.1) = \boxed{40\pi \text{ cm}^3}$

$\approx$ Error Max

(35) $\sin 33^\circ$

$x_1 = 30^\circ = \frac{\pi}{6}$

$\Delta x = 3^\circ = (3^\circ)\left(\frac{\pi}{180^\circ}\right) = \frac{\pi}{60}$

tangent line to $\sin x$ @ $x = \frac{\pi}{6}$:

$f(x) = \sin x \longrightarrow f\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$f'(x) = \cos x \longrightarrow f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$L_{\frac{\pi}{6}}(x) = f'\left(\frac{\pi}{6}\right)\left(x - \frac{\pi}{6}\right) + f\left(\frac{\pi}{6}\right)$

$= \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) + \frac{1}{2}$

$L_{\frac{\pi}{6}}\left(\frac{\pi}{6} + \frac{\pi}{60}\right) = \frac{\sqrt{3}}{2}\left(\frac{\pi}{60}\right) + \frac{1}{2}$

$\boxed{= \frac{\sqrt{3}\,\pi}{120} + \frac{1}{2}}$ or $\frac{\sqrt{3}\,\pi + 60}{120}$

$\approx \sin 33^\circ$

B6 $x^2 - 3xy + y^2 - 6 = x^2y^3 + 8$

$\Rightarrow 2x - 3y - 3xy' + 2yy' = 2xy^3 + 3x^2y^2y'$

$\Rightarrow -3xy' + 2yy' - 3x^2y^2y' = 2xy^3 - 2x + 3y$

$\Rightarrow y' = \dfrac{2xy^3 - 2x + 3y}{-3x + 2y - 3x^2y^2}$

$\Rightarrow y'\Big|_{(1, -\frac{13}{3})} = \dfrac{2(1)(-\frac{13}{3})^3 - 2(1) + 3(-\frac{13}{3})}{-3(1) + 2(-\frac{13}{3}) - 3(1)^2(-\frac{13}{3})^2} = m$

$\Rightarrow \boxed{y = m(x - 1) - \frac{13}{3}}$