MAT 201 Test 1, Fall, 2019 (5 pts) Name _____
100 Points Covers Chapter 1

Do all your work and put all your answers WITH your work, CIRCLED, on the white paper provided. All I want on this sheet is your NAME! Spend no more than 2 minutes on any single problem on your first pass through the test. If you don't finish a problem in 2 or 3 minutes, start a fresh sheet of paper for the next problem, and so on.

1. (15 pts) The point $P(3,12)$ lies on the graph of $f(x)=3x^2-5x$. Estimate the slope of this curve at *P*, by evaluating the slope between the point *P* and the point $Q(3.001,\ f(3.001))$.
2. (5 pts) Any guess as to what the actual slope is at $x = 3$?
3. (5 pts) Based on your answer to #2, write the equation of the tangent line to $f(x)$ at *P*.
4. (5 pts each) Evaluate the following limits, if they exist. If one does not exist, explain why.

   a. $\lim\limits_{x\to 5^+}\dfrac{3x^2-13x-10}{|x-5|}$ b. $\lim\limits_{x\to 5^-}\dfrac{3x^2-13x-10}{|x-5|}$ c. $\lim\limits_{x\to 5}\dfrac{3x^2-13x-10}{|x-5|}$

5. (15 pts) Sketch the graph of the piecewise-defined function $f(x)=\begin{cases}\dfrac{3}{x+2} & \text{if } x<1\\ x^2+1 & \text{if } x\ge 1\end{cases}$. I'm looking for key points labeled with ordered-pair labels (OPL's, baby!)

Bonus (5 pts) On what interval(s) is f in #5 continuous?

6. Simplify $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ for the following functions:
   a. (10 pts) $f(x)=3x^2-5x$
   b. (5 pts) $f(x)=\dfrac{1}{x}$
7. (10 pts) Sketch a plausible graph of a mostly-smooth function f that has the following properties. (Note: That very last condition is a later topic, "limits *at* infinity." So, 5 bonus points for the horizontal asymptote.)
   a. $\lim\limits_{x\to -3^-} f(x)=\infty$ d. $\lim\limits_{x\to 2^-} f(x)=3$ g. $f(3)=0$
   b. $\lim\limits_{x\to -3^+} f(x)=-\infty$ e. $\lim\limits_{x\to 2^+} f(x)=-1$ h. $\lim\limits_{x\to 8} f(x)=\infty$
   c. $f(-1)=0$ f. $f(2)=3$ i. $\lim\limits_{|x|\to\infty} f(x)=1$
8. (10 pts) Prove that $\lim\limits_{x\to 3}(5x+2)=17$, using the $\varepsilon-\delta$ definition of limit.
9. (5 pts) Prove that the equation $\dfrac{1}{4}\cdot 2^x-x^2+4x-3=0$ has a root in the interval $(0,2)$, but *do not solve*!

BONUS SECTION: Work any 2 bonus questions for up to 10 bonus points.

1. (5 pts) Prove that $\lim\limits_{x\to 2}(3x^2-5x+1)=3$
2. (5 pts) Evaluate $\lim\limits_{h\to 0}\dfrac{\sqrt{2x+2h}-\sqrt{2x}}{h}$, if it exists. If it does not, state why.
3. (5 pts) See if you can *squeeze* out a *convincing* argument to support the statement $\lim\limits_{x\to 0}\left(x^2\sin\left(\dfrac{\pi}{x}\right)\right)=0$.