201 TEST 1 FALL '19

① $f(x) = 3x^2 - 5x$, $P(3, 12)$, $Q(3.001, f(3.001))$

15pts

$$\frac{f(3.001) - f(3)}{3.001 - 3} = \frac{3(3.001)^2 - 5(3.001) - 12}{.001}$$

$$= \frac{3(9.006001) - 5(3.001) - 12}{.001} = 27.018003 - 15.005 - 12$$

$$= \frac{.013003}{.001} = \boxed{13.003 = m}$$

② 5pts $\boxed{m = 13}$

③ 5pts $\boxed{y = 13(x-3) + 12}$ is tan. line

④ ⓐ 5pts $\frac{3x^2 - 13x - 10}{|x-5|} = \frac{3x^2 - 13x - 10}{x-5}$ (if $x > 5$)

$$= \frac{(3x+2)(x-5)}{(x-5)} = 3x+2 \xrightarrow{x \to 5^+} \boxed{17}$$

ⓑ $x < 5 \longrightarrow \ldots$ $\frac{(3x+2)(x-5)}{-(x-5)} = -3x-2 \xrightarrow{x \to 5^-} \boxed{-17}$

ⓒ $\lim_{x \to 5^+} f(x) = 17 \neq 17 = \lim_{x \to 5^-} f(x) \rightarrow \boxed{\lim_{x \to 5} f(x) \nexists}$

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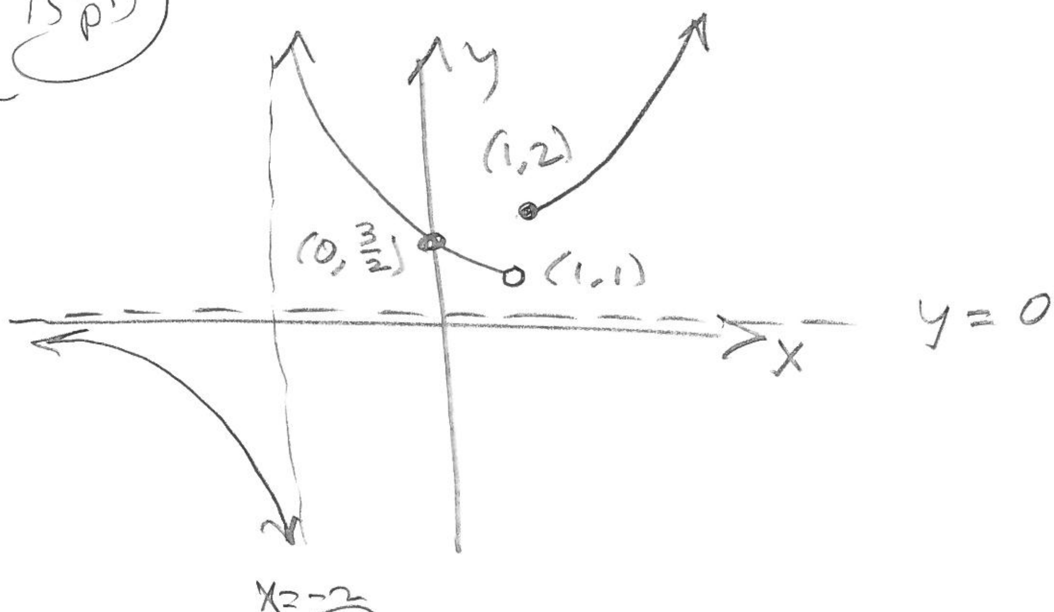
5 $\quad$ 15 pts

$$f(x)=\begin{cases} \dfrac{3}{x+2} & \text{if } x<1 \\ x^2+1 & \text{if } x\geq 1 \end{cases} \qquad \begin{aligned} \frac{3}{1+2}&=1 \\ 1^2+1&=2 \end{aligned}$$



$\boxed{\text{Bonus}}$ 5 pts f is cnt^s on $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$

We say that f is cnt^s from the right @ $x=1$

6 $\quad$ 10 pts $\quad f(x)=3x^2-5x \longrightarrow \dfrac{f(x+h)-f(x)}{h}$

a

$$=\frac{3(x+h)^2-5(x+h)-[3x^2-5x]}{h}=\frac{3x^2+6xh+3h^2-5x-5h-3x^2+5x}{h}$$

$$=\frac{6xh+3h^2-5h}{h}=\frac{h(6x+3h-5)}{h}=6x+3h-5 \quad (h\neq 0)$$

$$\xrightarrow{h\to 0} \boxed{6x-5}$$

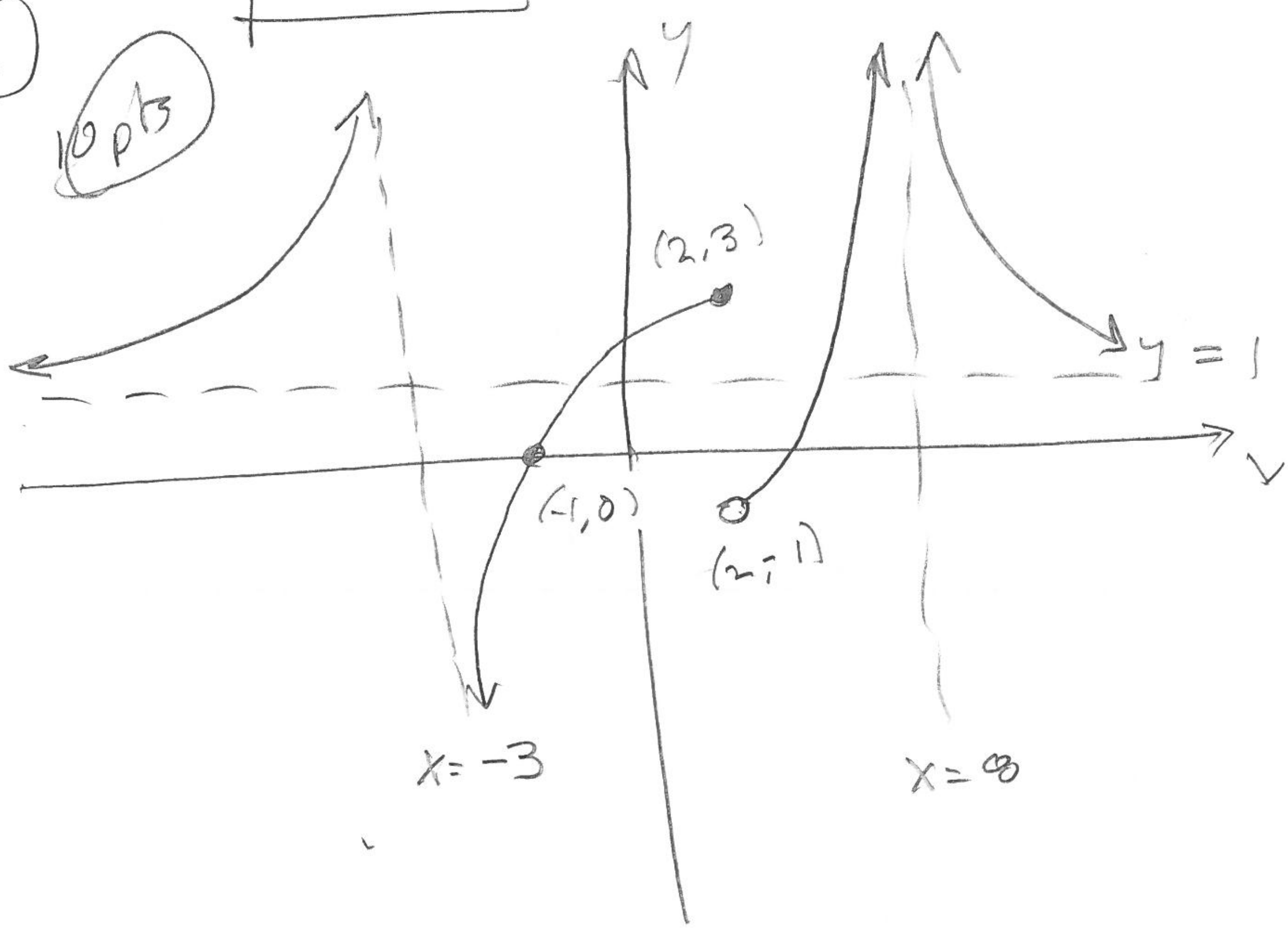
6b 5 pts $f(x)=\frac{1}{x} \Longrightarrow \frac{f(x+h)-f(x)}{h}$

$$= \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{1}{h}\left[\frac{1}{x+h}-\frac{1}{x}\right]$$

$$= \frac{1}{h}\left[\frac{x-(x+h)}{x(x+h)}\right] = \frac{1}{h}\left[\frac{-h}{x(x+h)}\right] = \frac{-1}{x(x+h)}$$

$$\xrightarrow{h\to 0} \boxed{-\frac{1}{x^2}}$$

7 10 pts



8 10 pts $\lim_{x \to 3} (5x+2) = 17$

Proof Let $\varepsilon > 0$. Define $\delta = \frac{\varepsilon}{5}$. Then

$0 < |x-3| < \delta \Rightarrow |(5x+2) - 17|$

$= |5x-15| = 5|x-3| < 5\delta = 5 \cdot \frac{\varepsilon}{5} = \varepsilon$ ∎

9 $\frac{1}{4} \cdot 2^x - x^2 + 4x - 3 = f(x)$

$\rightarrow f(0) = \frac{1}{4} - 0^2 + 4(0) - 3 = \frac{1}{4} - \frac{12}{4} = -\frac{11}{4} < 0$

$f(2) = \frac{1}{4} \cdot 2^2 - 2^2 + 4(2) - 3$

$= \frac{1}{4} \cdot 4 - 4 + 8 - 3 = 9 - 7 = 2 > 0$

f(x) is cnt's, $f(0) = -\frac{11}{4} < 0 < 2 = f(2)$

$\Rightarrow \exists\, c \in (0,2) \ni f(c) = 0$, by IVT on $[0,2]$. ∎

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B1 5pts

Claim $\lim_{x\to 2}(3x^2-5x+1)=3$

lim

Scratch $|3x^2-5x+1-3| = |3x^2-5x-2|$

$= |3x+1||x-2| < |3x+1|\delta$

Need bound on $3x+1$?

Assume $\delta \le 1$. Then $1<x<3$

$\Rightarrow 3 < 3x < 9$

$\Rightarrow 4 < 3x+1 < 10 \Rightarrow |3x+1| < 10$!

Proof Let $\varepsilon > 0$. Define $\delta = \min\{1, \frac{\varepsilon}{10}\}$.

Then $0 < |x-2| < \delta \Rightarrow |(3x^2-5x+1)-3|$

$= |3x^2-5x-2| = |3x+1||x-2| < 10|x-2|$

$< 10\delta \le 10\cdot\frac{\varepsilon}{10} = \varepsilon$ ∎

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B2 5pts $\dfrac{\sqrt{2x+2h}-\sqrt{2x}}{h}$

$$= \left(\frac{\sqrt{2x+2h}-\sqrt{2x}}{h}\right)\left(\frac{\sqrt{2x+2h}+\sqrt{2x}}{\sqrt{2x+2h}+\sqrt{2x}}\right)$$

$$= \frac{2x+2h-2x}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)} = \frac{2h}{h\left(\sqrt{2x+2h}+\sqrt{2x}\right)}$$

$$= \frac{2}{\sqrt{2x+2h}+\sqrt{2x}} \xrightarrow{h\to 0} \frac{2}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}}$$

Check $(2x)^{\frac{1}{2}} \mapsto \frac{1}{2}(2x)^{-\frac{1}{2}}(2)$ ✓

B3 5pts $-1 \leq \sin\left(\frac{\pi}{x}\right) \leq 1$

$$\Rightarrow \quad -x^2 \leq x^2 \sin\left(\frac{\pi}{x}\right) \leq x^2$$

$$\xrightarrow{x\to 0} \quad 0 \leq \lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right) \leq 0$$

$$\Rightarrow \lim_{x\to 0} x^2 \sin\left(\frac{\pi}{x}\right) = 0 \quad \blacksquare$$

By Squeeze Theorem.