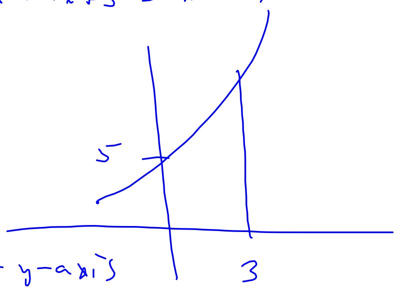
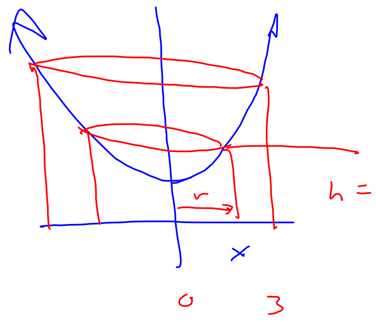
Shell -vs- Washers

Revolve $y = x^2+4x+5$ about the y-axis from $x=0$ to $x=3$

$x^2+4x+5 = x^2+4x+2^2-4+5 = (x+2)^2+1$



about y-axis



$2\pi r h \, dx$

$2\pi \int y r \, dx = 2\pi \int f(x)$

$= 2\pi \int_0^3 x f(x) \, dx$

(x^2+4x+5)

$= 2\pi \int_0^3 x(x^2+4x+5)\, dx$

$x \, dx$

$= 2\pi \int_0^3 (x^3+4x^2+5x)\, dx$

$= 2\pi \left[\frac{x^4}{4} + \frac{4}{3}x^3 + \frac{5}{2}x^2 \right]_0^3$

$= 2\pi \left[\frac{3^4}{4} + \frac{4}{3}(3^3) + \frac{5}{2}(3^2) \right] - 0$

$= 2\pi \left[\frac{81}{4} + 36 + \frac{45}{2} \right]$

$= 2\pi \left[\frac{81+144+90}{4} \right]$

$= \pi \left[\frac{225+90}{2} \right] = \frac{315\pi}{2}$

$\int fg \neq \int f \int g$

$\sum xy \neq \sum x \sum y$

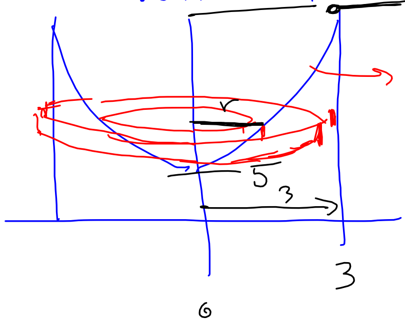
$\sum_{k=1}^{2} x_k y_k = x_1 y_1 + x_2 y_2$

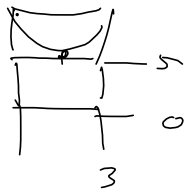
$\left(\sum_{k=1}^{2} x_k\right)\left(\sum_{k=1}^{2} y_k\right)$

$= (x_1+x_2)(y_1+y_2)$

$= x_1 y_1 + x_1 y_2 + x_2 y_1 + x_2 y_2$

Washer Method





$x^2+4x+5 = f(x)$

$f(0) = 5$

$3^2+4(3)+5$

$= 9+12+5 = 26$

$$\pi\int_5^{26}\left(\text{outer}^2 - \text{inner}^2\right)dy$$

Inner takes work!

$f(x) = (x+2)^2+1 = y$

$\Rightarrow (x+2)^2 = y-1$

$x+2 = \pm\sqrt{y-1}$

$x = -2 \pm \sqrt{y-1}$

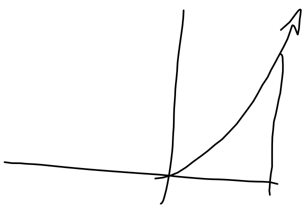
$x = -2 + \sqrt{y-1}$

$$= \pi\int_5^{26}\left(3^2 - \left(-2+\sqrt{y-1}\right)^2\right)dy$$

$$+ \pi r^2 h = \pi\int \ldots + 45\pi$$

$\pi(3)^2(5)$ For the lower, right, circular cylinder.

On final over this stuff, I'll make it so it's one integral. No "$+45\pi$" added on



Something like this

Section 5.5-type question

Do MVT for integrals on quadratic.

$$f_{AVG} = \frac{1}{b-a} \int_a^b \text{quadratic}\, dx$$

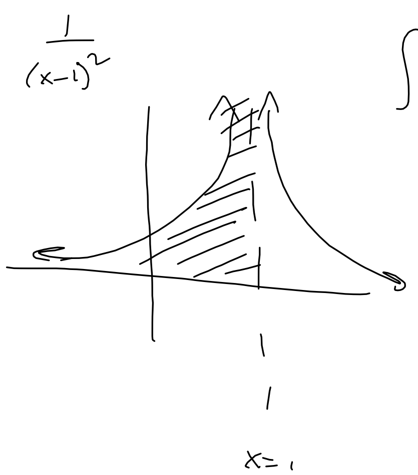
Followup:

set quadratic $= c$, where $f(c) = f_{AVG}$.

MVT: f cnt^s on $[a,b]$ & difbl on (a,b) $\Rightarrow$ $\exists\, c \in (a,b) \ni f'(c) = m_{AVG}$.

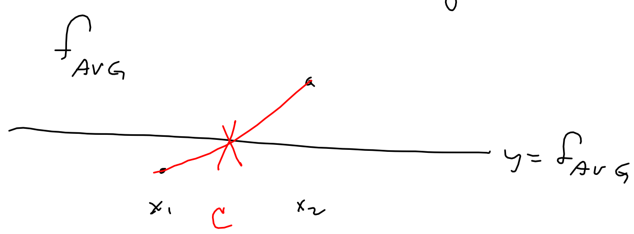
MVT Integrals: f cnt^s on $[a,b]$

$$f_{AVG} = \frac{1}{b-a} \int_a^b f(x)\, dx$$



$\int_0^1 \frac{dx}{(x-1)^2}$ Doesn't hold, b/c $\frac{1}{(x-1)^2}$ not cnt^s @ $x=1$.

Finding the c for MVT I isn't part of the theorem, but a consequence of IVT



c $\exists$ b/c IVT.

$$\frac{d}{dx}\int_0^{x} \sin^3\left(t^2\cos(t)\right)\,dt$$

$$= \sin^3\left(x^2 (\cos(x))\right)$$

A.L.

$$\frac{d}{dx}\int_0^{\tan x} \sin^3\left(t^2\cos(t)\right)\,dt$$

$$= \sin^3\left(\tan^2(x)\cos\left(\tan(x)\right)\right)\cdot\sec^2 x$$

$$\int_a^b = -\int_b^a$$

$$\frac{d}{dx}\left[\int_{x^2}^{\tan x}\sin^3\left(t^2\cos(t)\right)\,dt\right]$$

$$= \frac{d}{dx}\left[\int_{x^2}^{0} + \int_0^{\tan x}\right]$$

$$\int_a^b = \int_a^c + \int_c^b$$

$$= \frac{d}{dx}\left[-\int_0^{x^2} + \int_0^{\tan x}\right]$$

$$= -\sin^3\left((x^2)^2\cos(x^2)\right)(2x)$$

$$+ \sin^3\left(\tan^2 x\,\cos(\tan x)\right)\cdot\sec^2 x$$

Alex got me twice, today on really nit-picky and petty stuff.

GOLD STAR