§5.5 #4

$g(t) = \frac{t}{\sqrt{t^2+5}}$ on $[2,5]$

Mills followup: He's an idiot.

Find m_{avg}.

Find $c \in (2,5)$ $\ni$ $f(c) = f_{ave}$

$\frac{1}{5-2} \int_2^5 \frac{t}{\sqrt{t^2+5}}\, dt$

$\boxed{\begin{aligned} u &= t^2+5 \\ du &= 2t\, dt \end{aligned}}$ ALWAYS SHOW THIS.

Shortest Possible

M1

$\frac{1}{3} \cdot \frac{1}{2} \int_2^5 (t^2+5)^{-\frac{1}{2}} (2t\, dt)$

$= \frac{1}{6} \left[\frac{(t^2+5)^{\frac{1}{2}}}{\frac{1}{2}} \right]_2^5 = \frac{1}{3} \left[(t^2+5)^{\frac{1}{2}} \right]_2^5$

M2

$dt = \frac{du}{2t}$

$\frac{1}{3} \int_2^5 u^{-\frac{1}{2}} \left(t \cdot \frac{du}{2t}\right)$

$= \frac{1}{6} \int_{t=2}^{t=5} u^{-\frac{1}{2}} du$

$u = t^2+5$

$t=2 \Rightarrow u = 2^2+5 = 9 = u$

$t=5 \Rightarrow u = 5^2+5 = 30 = u$

$\frac{1}{6} \int_9^{30} u^{-\frac{1}{2}} du$

$= \frac{1}{6} \left[2u^{\frac{1}{2}} \right]_9^{30} = \frac{1}{3} \left[30^{\frac{1}{2}} - 9^{\frac{1}{2}} \right]$

$= \frac{1}{6} \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_{t=2}^{t=5}$

$2 \,|\, 30$

$3 \,|\, 15$

5

$= \frac{1}{3} \left[(t^2+5)^{\frac{1}{2}} \right]_2^5$

$= \frac{1}{3} \left[30^{\frac{1}{2}} - 9^{\frac{1}{2}} \right]$

$\boxed{= \frac{1}{3} \left[\sqrt{30} - 3 \right]}$

$= \frac{\sqrt{30} - 3}{3}$

$= \frac{\sqrt{30}}{3} - 1$

Intermediate Algebra Training:

$3x = 7$

$\frac{3x}{3} = \frac{7}{3}$

$x = \frac{7}{3}$

$\boxed{\begin{aligned} -3x &< 7 \\ x &> \frac{7}{-3} \end{aligned}}$

Efficient, complete, true.

Followup (Not part of #4, but a standard thing to look for.)

Set $f(t) = f_{avg}$ on $[2,5]$:

$$\frac{t}{\sqrt{t^2+5}} = \frac{\sqrt{30}}{3} - 1 \qquad LCD = 3\sqrt{t^2+5}$$

$$\frac{t}{\sqrt{t^2+5}} \cdot \frac{3}{3} = \left(\frac{\sqrt{30}-3}{3}\right) \frac{\sqrt{t^2+5}}{\sqrt{t^2+5}}$$

$$\frac{3t}{LCD} = \frac{(\sqrt{30}-3)\sqrt{t^2+5}}{LCD}$$

$\boxed{>, <, \text{Keep LCD}}$

" = " Throw LCD away

$$(\quad)^2 = (\qquad)^2$$

$$9t^2 = (\sqrt{30}-3)^2 (t^2+5)$$

$$9t^2 - (\sqrt{30}-3)^2 t^2 - (\sqrt{30}-2)^2 \cdot 5$$

Let $d = (\sqrt{30}-3)^2$

$$9t^2 - dt^2 - 5d \overset{SET}{=} 0$$

$a = 9$, $b = -d$, $c = -5d$

$$\begin{array}{r} {}^{3}36 \\ 5 \\ \hline 180 \end{array}$$

$$b^2 - 4ac = (-d)^2 - 4(9)(-5d)$$
$$= d^2 + 180d$$

$$t = \frac{d \pm \sqrt{d^2+180d}}{2(9)}$$

Save the d in your calculator.
Use recall to plug in.

$\Rightarrow t \approx$ 3.273505511

$$\frac{t}{\sqrt{t^2+5}} = \frac{\sqrt{30}}{3} - 1$$

Clear Fractions Method (Doesn't work for inequalities

$LCD = 3\sqrt{t^2+5}$

$$3t = \left(\frac{\sqrt{30}}{3} - 1\right)\left(\sqrt{t^2+5}\right)(3)$$

New "d"

$$3t = (\sqrt{30} - 3)\sqrt{t^2+5}$$

$d = \sqrt{30} - 3)$

$$9t^2 = d^2(t^2+5)$$

$$9t^2 - d^2t^2 - 5d^2 = 0$$

$$(9 - d^2)t^2 = 5d^2$$

$$t^2 = \frac{5d^2}{9-d^2}$$

$$t = \pm\sqrt{\frac{5d^2}{9-d^2}}$$

$$c = \sqrt{\frac{5d^2}{9-d^2}}$$