Test 3 Trig eq'n Take-Home

$\sin x + \sqrt{3}\cos x + 1 = 0$

$\sin x + 1 = -\sqrt{3}\cos x$

$(\quad)^2 = (\quad)^2$

$\Rightarrow \sin^2 x + 2\sin x + 1 = (-\sqrt{3})^2\cos^2 x = 3\cos^2 x = 3 - 3\sin^2 x$ ← $3(1-\sin^2 x)$

$4\sin^2 x + 2\sin x - 2 = 0$

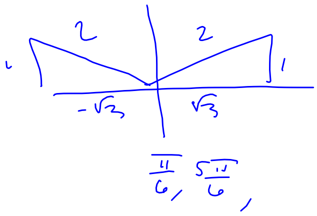
$2\sin^2 x + \sin x - 1 = 0$

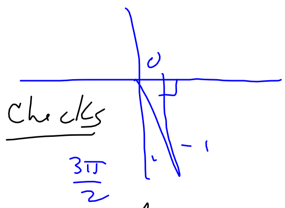
$a = 2,\ b = 1,\ c = -1$

$b^2 - 4ac = 1^2 - 4(2)(-1) = 1 + 8 = 9$

$x = \dfrac{-1 \pm \sqrt{9}}{2(2)} = \dfrac{-1 \pm 3}{4}$ → $\dfrac{2}{4} = \dfrac{1}{2}$; → $-\dfrac{4}{4} = -1$

$\sin x = \dfrac{1}{2}$ $\qquad$ $\sin x = -1$





Squaring Both sides casts a net, but some fish may need to be released.

$\sin\frac{\pi}{6} + \sqrt{3}\cos\frac{\pi}{6} + 1$

$= \frac{1}{2} + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right) + 1$

$= \frac{1}{2} + \frac{3}{2} + 1 = 3$

$x = \frac{\pi}{6}$ doesn't check

$a = b$

$\Rightarrow a^2 = b^2$, but

$a^2 = b^2 \not\Rightarrow a = b$,

so we check.

$\sin\frac{5\pi}{6} + \sqrt{3}\cos\frac{5\pi}{6} + 1$

$= \frac{1}{2} - \frac{3}{2} + 1 = 0$ ✓

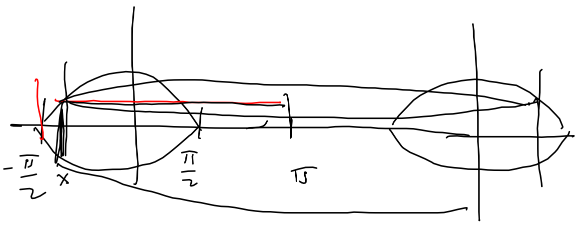
So $\frac{\pi}{6}$ was an extraneous root.

Be aware of this whenever you square both sides of an equation.

$a = b \Rightarrow a^2 = b^2$, but

$a^2 = b^2 \not\Rightarrow a = b$, b/c:

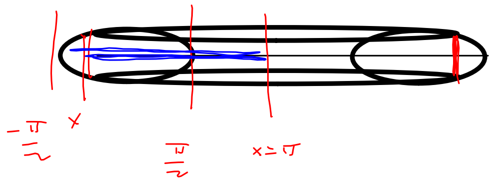
$(-3)^2 = 3^2$, but $\boxed{-3 \neq 3}$



$6\cos^4 x$

$-6\cos^4 x$

$2\pi r h\, dx$



$$2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\pi-x)(12\cos^4 x)\,dx$$

$$=2\pi\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(12\pi\cos^4 x-12\pi x\cos^4 x\right)dx$$

ODD —

EVEN +

$12\pi(-)(+)=-$ ODD

$$=2\pi\cdot 2\int_0^{\frac{\pi}{2}}12\pi\cos^4 x\,dx$$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$$=48\pi^2\int_0^{\frac{\pi}{2}}$$

$\cos^4 x=(\cos^2 x)^2$

$$=\frac{48\pi^2}{8}\int_0^{\frac{\pi}{2}}\left(3+4\cos(2x)+\cos(4x)\right)dx$$

$=\left(\dfrac{1+\cos 2\theta}{2}\right)^2$

$$=6\pi^2\left[3x+2\sin(2x)+\frac{1}{4}\sin(4x)\right]_0^{\frac{\pi}{2}}$$

$=\frac{1}{4}\left[1+2\cos 2\theta+\cos^2(2\theta)\right]$

$$=6\pi^2\left[\frac{3\pi}{2}+0+0\right]=9\pi^3$$

$=\frac{1}{4}\left[1+2\cos(2\theta)+\left(\dfrac{1+\cos(4\theta)}{2}\right)\right]$

$4\cos(2x)\,dx$

$=2\cos(2x)\cdot 2\,dx$

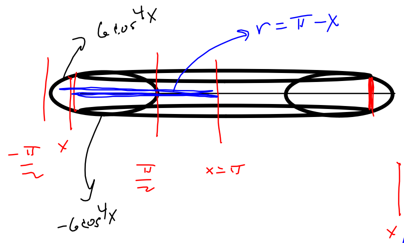
$=\frac{1}{4}+\frac{1}{2}\cos(2\theta)+\frac{1}{8}+\frac{1}{8}\cos 4\theta$

$=\frac{3}{8}+\frac{1}{2}\cos(2\theta)+\frac{1}{8}\cos(4\theta)$

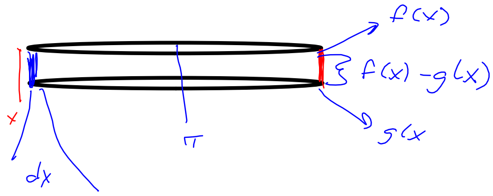
Final Answer!

$=\frac{1}{8}\left[3+4\cos(2\theta)+\cos(4\theta)\right]$

(7) We find the volume of the solid of revolution obtained when the region enclosed by $f(x) = 6\cos^4 x$ & $g(x) = -6\cos^4 x$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ is revolved about the line $x = \pi$



$2\pi r h\,dx$



So, volume $= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\pi - x)\left[6\cos^4 x - (-6\cos^4 x)\right] dx$

$= 2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\pi - x)(12\cos^4 x)\,dx = (2\pi)(12) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\pi - x)\cos^4 x\,dx$

$= 48\pi \left[\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \pi \cos^4 x\,dx - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x\cos^4 x\,dx \right]$

Even func on symmetric interval

ODD Func on symmetric interval

$= 2\pi \cdot 48\pi \int_0^{\frac{\pi}{2}} \cos^4 x\,dx$

$= 2\cdot 48\pi^2 \int_0^{\frac{\pi}{2}} \left(\frac{1+\cos(2x)}{2}\right)^2 dx$

$= \frac{2\cdot 48\pi^2}{4} \int_0^{\frac{\pi}{2}} \left(\cos^2(2x) + 2\cos(2x) + 1\right) dx$

$= 2\cdot 12\pi^2 \int_0^{\frac{\pi}{2}} \left(\frac{1+\cos(4x)}{2} + 2\cos(2x) + 1\right) dx$

$= 24\pi^2 \left[\int_0^{\frac{\pi}{2}} \frac{1}{2}\,dx + \int_0^{\frac{\pi}{2}} \frac{1}{2}\cos(4x)\,dx + \int_0^{\frac{\pi}{2}} 2\cos(2x) + \int_0^{\frac{\pi}{2}} 1\,dx \right]$

$= 24\pi^2 \left[\int_0^{\frac{\pi}{2}} \frac{3}{2}\,dx + \frac{1}{2}\cdot\frac{1}{4}\int_0^{\frac{\pi}{2}} \cos(4x)\cdot 4\,dx + 2\cdot\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos(2x)\cdot 2\,dx \right]$

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