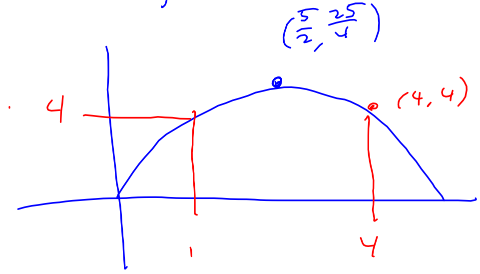
$y = -x^2+5x$ about $x=1$ from $x=1$ to $x=4$

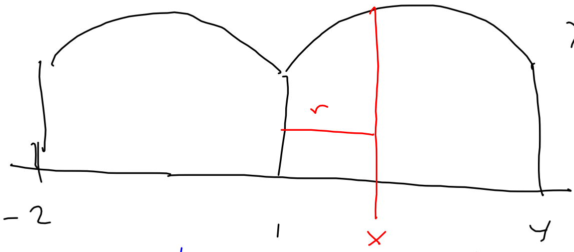


$$y = -\left(x-\frac{5}{2}\right)^2+\frac{25}{4}$$

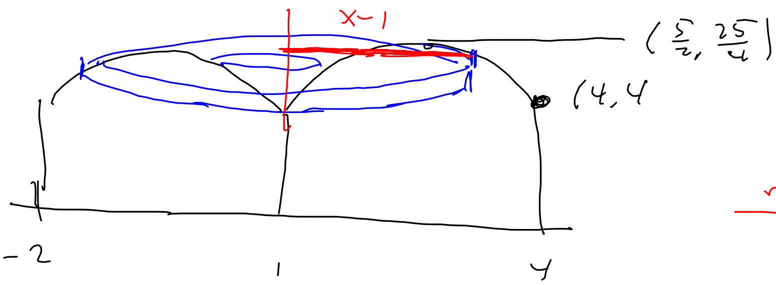
$\Rightarrow \ldots$

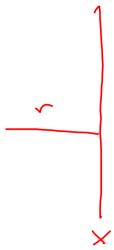
$$x = \frac{5}{2} \pm \sqrt{\frac{25}{4}-y}$$

$$x = \frac{5}{2} \pm \sqrt{-\left(y-\frac{25}{4}\right)}$$



$$2\pi\int_a^b r\,h\,dx = 2\pi\int_1^4 (x-1)(-x^2+5x)\,dx = \frac{99\pi}{2}$$





$$x = \frac{5}{2} \pm \sqrt{-\left(y-\frac{25}{4}\right)}$$

$$\Rightarrow \underline{\text{Radius}} = \frac{5}{2} \pm \sqrt{-\left(y-\frac{25}{4}\right)} - 1$$

$$\pi\int_4^{\frac{25}{4}} \left(\text{outer}^2-\text{inner}^2\right) dy$$

$$= \pi\int_4^{\frac{25}{4}} \left(\left(\frac{3}{2}+\sqrt{-\left(y-\frac{25}{4}\right)}\right)^2 - \left(\frac{3}{2}-\sqrt{-\left(y-\frac{25}{4}\right)}\right)^2\right) dy$$

$$+\ \pi\cdot 3^2\cdot 4 \quad = \frac{99\pi}{2}$$

cylinder under $y=4$

$4\sqrt{2x}$ & $2x^2$ about y-axis

Shell method easier when functions of x about y-axis.

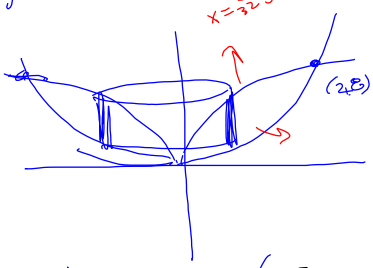
$4\sqrt{2x} = 2x^2$

$16(2x) = 4x^4$

$4x^4 - 32x = 0$

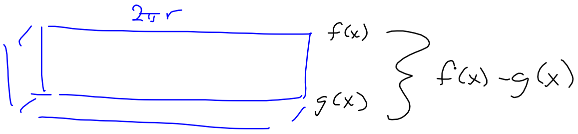
$4x(x^3 - 8) = 0$

$x = 0, 2$



$2\pi r h \Delta r = 2\pi x \left(4\sqrt{2x} - 2x^2\right) dx$

$V = 2\pi \int_0^2 x\left(4\sqrt{2x} - 2x^2\right) dx$



Washer Method

Disc: $\pi r^2 h = \pi r^2 dy$

$\pi(\text{outer}^2 - \text{inner}^2)\, dy$

$y = 4\sqrt{2x} = y$

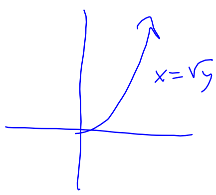
$16(2x) = y^2$

$x = \frac{1}{32}y^2$ = inner

$y = 2x^2 = y$

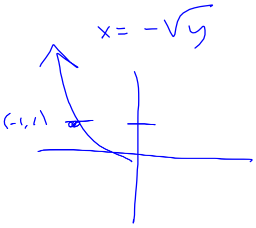
$x^2 = \frac{1}{2}y$

$x = \pm\sqrt{\frac{y}{2}} = \pm\frac{\sqrt{2y}}{2}$



We want

$\pi r^2 h$ $\quad x = \frac{\sqrt{2y}}{2} = \frac{1}{\sqrt{2}}\sqrt{y}$



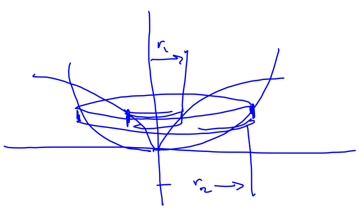
$\pi \int_a^b (f(y))^2 dy$ Disc

Washer:

$\pi \int_a^b (\text{outer}^2 - \text{inner}^2) dy$

$= \pi \int_0^8 (r_2^2 - r_1^2) dy$

$$\boxed{= \pi \int_0^8 \left(\left(\sqrt{\frac{y}{2}}\right)^2 - \left(\frac{y^2}{32}\right)^2\right) dy}$$



§5.4 Work

$F = ma$

work $= Fd$

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§5.5 Mean Value Thm for Integrals

MVT: f is cnt's on $[a,b]$ & d.f.b'l on (a,b)

$\Rightarrow \exists\, c \in (a,b) \ni f'(c) = m_{AVG}$

$= \dfrac{f(b)-f(a)}{b-a}$.

$\boxed{\text{MVT for Integrals:}}$

f cnt's on $[a,b]$ then the average value for f on $[a,b]$ is

$$\frac{1}{b-a}\int_a^b f(x)\,dx \qquad \Sigma \Delta x = b-a \,!$$

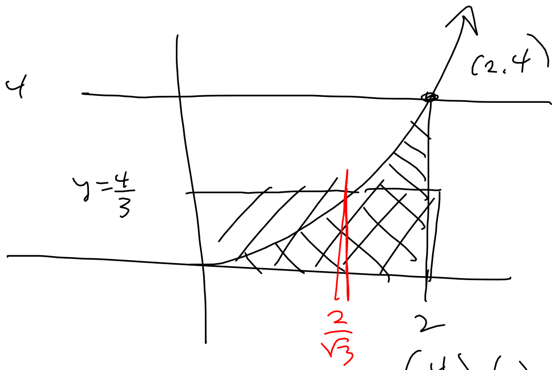
Tricky questions for BOTH MVTs:

Find c where $f'(c) = m_{avg}$.

Find c where $f(c) = f_{AVG}$.

Find avg of $f(x) = x^2$ on $[0,2]$

$$\frac{1}{2}\int_0^2 x^2\,dx = \frac{1}{2}\left(\frac{x^3}{3}\right)\Bigg]_0^2 = \frac{1}{2}\left[\frac{8}{3}\right] = \frac{4}{3}$$



$$\left(\frac{4}{3}\right)(2) = \frac{8}{3}$$

$x^2 \overset{SET}{=} \frac{4}{3}$

$x = \pm\frac{2}{\sqrt{3}} \Rightarrow c = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \in (0,2)$