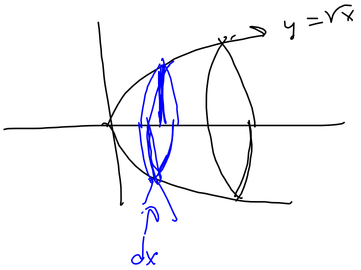
Disk Method (Washer Method)



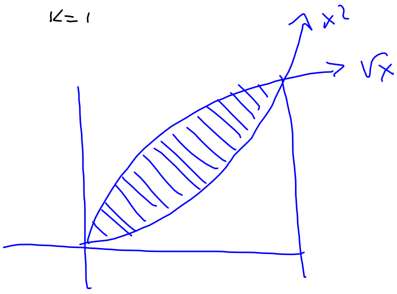
Rotate $f(x)=\sqrt{x}$ from $x=0$ to $x=4$ about the x-axis

Area of disk (short, squat cylinder)

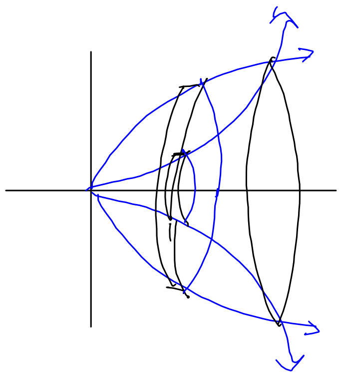
$\pi r^2 h = \pi f(x)^2 \Delta x$

Add 'em up:

$$\sum_{k=1}^{n} \pi f(x_k)^2 \Delta x \xrightarrow{n \to \infty} \int_0^4 \pi (\sqrt{x})^2 dx$$



Revolve the shaded region about the x-axis



Basically revolve x^2 & $\sqrt{x}$ separately & subtract off the inner one.

$$\pi \int_a^b (\text{outer})^2 dx - \pi \int_a^b (\text{inner})^2 dx$$

$$= \pi \int_a^b (\text{outer}^2 - \text{inner}^2)\, dx$$

$$V = \pi \int_0^4 \left((\sqrt{x})^2 - (x^2)^2\right) dx = \pi \int_0^4 (x - x^4)\, dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^4$$

$4 \rightarrow 1$, idiot.

$$= \frac{4^2}{2} - \frac{4^5}{5} = 8 - \frac{1024}{5}$$

$\sqrt{x^2} = |x|$

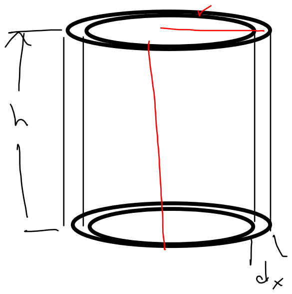
$(\sqrt{x})^2 = x$!

$$V = \pi \int_0^1 \left((\sqrt{x})^2 - (x^2)^2\right) dx = \pi \left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \pi\left(\frac{1}{2} - \frac{1}{5}\right) =$$

$$\pi \left(\frac{5-2}{10}\right) = \frac{3\pi}{10} =$$

Shell method.

what is the volume of a cylinder wall of height h, thickness w, radius r?



we'd do an inner radius & outer radius & subtract the volume of the cylinders.

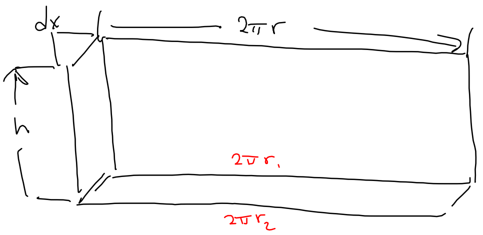
r_1 = inner

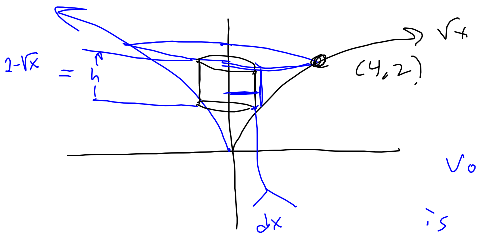
r_2 = outer.

$\pi r_2^2 h - \pi r_1^2 h$

Here's an approximate way that approaches the actual as $\Delta x \to 0$.

Slice vertically & lay flat.





Revolve $y=\sqrt{x}$ about the y-axis, from $x=0$ to $x=4$

Vol of the shell is $2\pi r h \Delta x = 2\pi x(2-\sqrt{x})\Delta x$

$$2\pi\int_0^4 x(2-\sqrt{x})\,dx$$

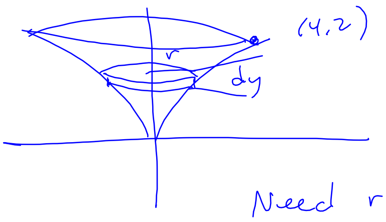
$x\sqrt{x} = x^1 x^{\frac{1}{2}} = x^{\frac{3}{2}}$

$$= 2\pi\int_0^4 \left(2x - x^{\frac{3}{2}}\right)dx = 2\pi\left[x^2 - \frac{2}{5}x^{\frac{5}{2}}\right]_0^4$$

$$= 2\pi\left[16 - \frac{2}{5}\cdot 32\right] = 2\pi\left[\frac{80-64}{5}\right] = \frac{2\pi(16)}{5}$$

$$= \frac{32\pi}{5}$$

Disk Method



about the y-axis

$$\pi\int_0^2 (y^2)^2\,dy = \pi\left[\frac{1}{5}y^5\right]_0^2$$

$$= \pi\left[\frac{32}{5}\right]$$

$$= \frac{32\pi}{5}$$

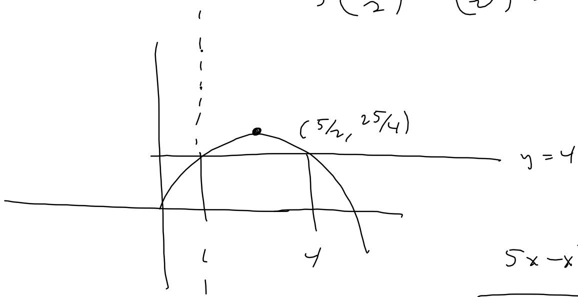
Need $r = g(y)$

$y = \sqrt{x}$

$r = y^2 = x$

$5x-x^2, \quad y=4, \quad \text{about } x=1$

$x(5-x) \qquad 5\left(\frac{5}{2}\right)-\left(\frac{5}{2}\right)^2 = \frac{25}{2}-\frac{25}{4}=\frac{25}{4}$

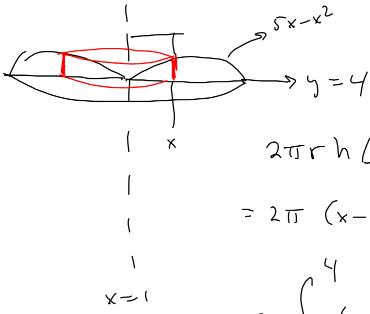


$5x-x^2=4$

$\boxed{-x^2+5x-4}=0$

$x^2-5x+4=0$

$(x-4)(x-1)=0$

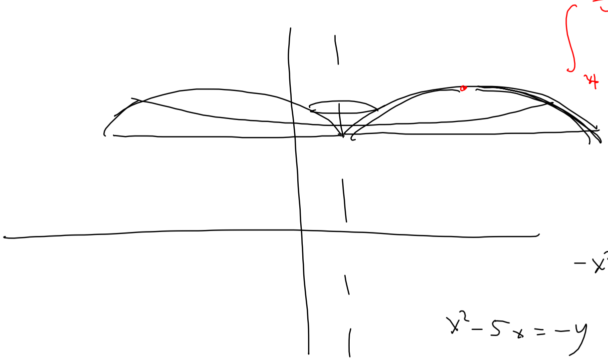


$2\pi r h \Delta x$

$= 2\pi\,(x-1)^2(5x-x^2-4)\,\Delta x$

$$2\pi\int_1^4 (x-1)^2(-x^2+5x-4)\,dx$$

$\int_4^{\frac{25}{4}}$



$y=5x-x^2$

$-x^2+5x=y$

$x^2-5x=-y$

$x^2-5x+\left(\frac{5}{2}\right)^2=-y+\frac{25}{4}$

$\left(x-\frac{5}{2}\right)^2=\frac{25}{4}-y$

$x-5/2=\pm\sqrt{\frac{25}{4}-y}$

$=\pm\sqrt{-\left(y-\frac{25}{4}\right)}$