$$\int \cos^{-5} x \sin x \, dx$$

$u = \cos x \Rightarrow du = -\sin x \, dx$

$\Rightarrow dx = \frac{du}{-\sin x}$

$$\int u^{-5} \sin x \frac{du}{-\sin x}$$

$$= -\int u^{-5} du = -\frac{u^{-4}}{-4} + C$$

$$= \frac{\cos^{-4} x}{4} + C$$

$u = 3x - 1$ → frequent mistake.

$u + 1 = 3x$

$\frac{u-1}{3} = x$

$$\int x^2 (3x-1)^5 dx \quad = \frac{1}{3}\int x^2 (3x-1)^5 (3dx)$$

$u = 3x - 1 =$

$\Rightarrow du = 3dx$

$dx = \frac{du}{3}$

$3x - 1 = u$

$3x = u + 1$

$x = \frac{u+1}{3}$

$x^2 = \left(\frac{u+1}{3}\right)^2 = \frac{u^2 + 2u + 1}{9}$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$

$(a+b)^5 = a^5 + 5a^4 b + 10a^3 b^2 + 10a^2 b^3 + 5ab^4 + b^5$

```
          1
        1   1
      1   2   1
    1   3   3   1
  1   4   6   4   1
1   5  10  10   5   1
```

$$\int x^2 (3x-1)^5 dx = \int \left(\frac{u^2+2u+1}{9}\right) u^5 \frac{du}{3}$$

$$= \frac{1}{27} \int u^5 (u^2 + 2u + 1) du$$

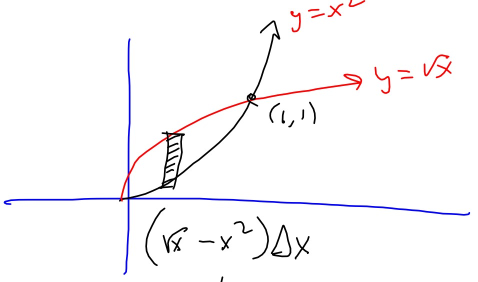
$$= \frac{1}{27} \int (u^7 + 2u^6 + u^5) du$$

$$\frac{1}{27}\left[\frac{u^8}{8} + \frac{2u^7}{7} + \frac{u^6}{6}\right] + C$$

$$= \frac{1}{27}\left[\frac{(3x-1)^8}{8} + \frac{2(3x-1)^7}{7} + \frac{(3x-1)^6}{6}\right] + C$$

§5.1

Area bdd by $y = x^2$, $y = \sqrt{x}$



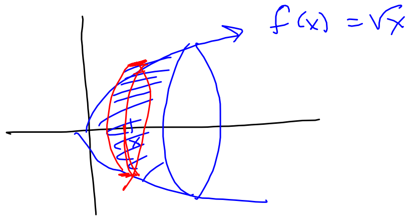
$(\sqrt{x} - x^2)\Delta x$

$$\text{Area} = \int_0^1 (x^{\frac{1}{2}} - x^2)\,dx = \int_a^b (\text{up} - \text{low})$$

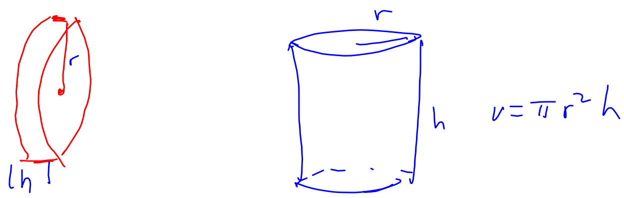
$$= \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1 = \frac{2}{3} - \frac{1}{3} = \boxed{\frac{1}{3} = \text{Area}}$$

$$= \left(\frac{2}{3}(1)^{\frac{3}{2}} - \frac{1}{3}(1)^3\right) - \left(\frac{2}{3}(0)^{\frac{3}{2}} - \frac{1}{3}(0)^3\right)$$

Find volume obtained by revolving $f(x) = \sqrt{x}$, from $x=0$ to $x=1$ about the x-axis



Volume of coin is area of face times thickness of coin.

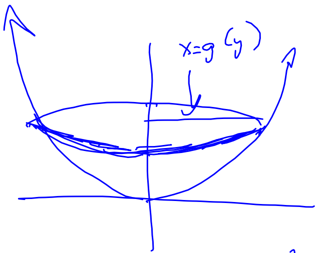


$\pi r^2 h = \pi r^2 \Delta x = \pi (f(x))^2 \Delta x$

$$\text{Volume} = \pi \int_a^b f(x)^2 dx = \pi \int_a^b y^2 dx$$ about x-axis.

about y-axis:

$$V = \pi \int_c^d g(y)^2 dy = \pi \int_c^d x^2 dy$$



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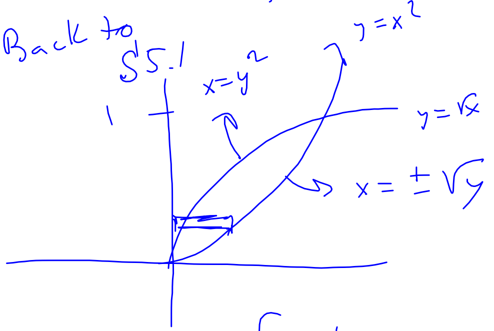


$x = \pm\sqrt{y} = \sqrt{y}$, by picture

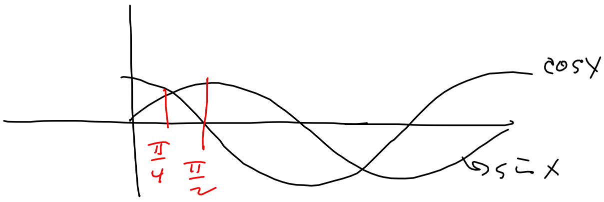
$\int g(y)\, dy$ version

$$\int \text{Right} - \text{Left} = \int_0^1 (\sqrt{y} - y^2)\, dy$$

§5.1 #9 $\int_0^{\frac{\pi}{2}} |2\sin x - 2\cos x|\,dx$

$\sin x = \cos x$

$\frac{\sin x}{\cos x} = 1$



$\int_0^{\pi} |f| = \int_0^{\frac{\pi}{4}} -f + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} f$

$= -\int_0^{\frac{\pi}{4}} (2\sin x - 2\cos x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (2\sin x - 2\cos x)\,dx$

$= -\left[-2\cos x - 2\sin x\right]_0^{\frac{\pi}{4}} + \left[-2\cos x - 2\sin x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= -\left(\left[-\frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right] - \left[-2\cos(0) - 2\sin(0)\right]\right)$

$+\left(\left[-2\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2}\right] - \left[-2\cos\frac{\pi}{4} - 2\sin\frac{\pi}{4}\right]\right)$

$= +\left(+\frac{4}{\sqrt{2}} + \left(-2(1) - 2\cdot 0\right)\right)$

$+\left(\left[-2\cdot 0 - 2\cdot 1\right] - \left[-2\left(\frac{1}{\sqrt{2}}\right) - 2\left(\frac{1}{\sqrt{2}}\right)\right]\right)$

$= \frac{4}{\sqrt{2}} - 4 + (-2) - \left[-\frac{2}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right]$

$= -6 + \frac{8}{\sqrt{2}} = -6 + 5.656\ldots$