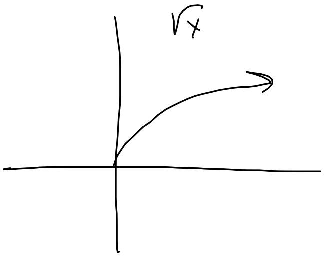
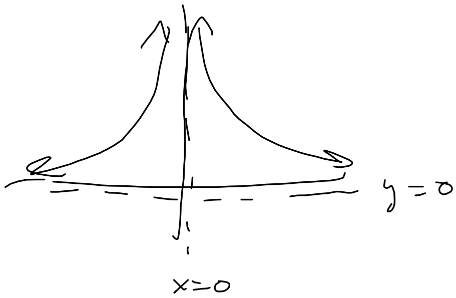
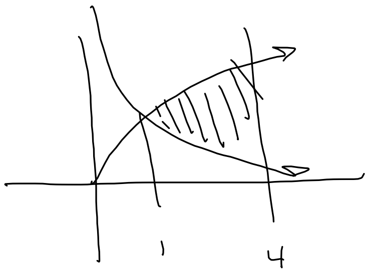
Find area bdd by $y=\sqrt{x}$ & $y=\frac{1}{x^2}$, & $x=1$ & $x=4$







$$\sqrt{x} = \frac{1}{x^2}$$

$$x = \frac{1}{x^4}$$

$$x^4 x = 1$$

$$\boxed{x^5 - 1 = 0}$$

$$(x-1)(x^4+x^3+x^2+x+1)$$

$$x^n - 1 = (x-1)(x^{n-1}+x^{n-2}+\dots+x^1+1)$$

$$x^{n-(n-1)} = x^1$$

$$x^5 = 1$$

$$\sqrt[5]{x^5} = \sqrt[5]{1}$$

$$x = 1$$

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & 1 & 0 \\ & x^4 & x^3 & x^2 & x & c & r \end{array}$$

$$(x-1)(x^4+x^3+x^2+x+1)$$

$$\int_1^4 (\text{upper} - \text{lower})\,dx$$

$$= \int_1^4 \left(\sqrt{x} - \frac{1}{x^2}\right)dx = \int_1^4 \left(x^{\frac{1}{2}} - x^{-2}\right)dx$$

$$= \left[\frac{2}{3}x^{\frac{3}{2}} + x^{-1}\right]_1^4 = \frac{2}{3}(4)^{\frac{3}{2}} + \frac{1}{4} - \left(\frac{2}{3}(1)^{3/2} + \frac{1}{1}\right)$$

$$\frac{2}{3}(2)^3 + \frac{1}{4} - \frac{2}{3} - 1 = \frac{16}{3} + \frac{1}{4} - \frac{2}{3} - 1 = \frac{14}{3} - \frac{3}{4} = \frac{56-9}{12} = \boxed{\frac{47}{12}}$$

$$x^{-2} \longrightarrow \frac{x^{-1}}{-1}$$

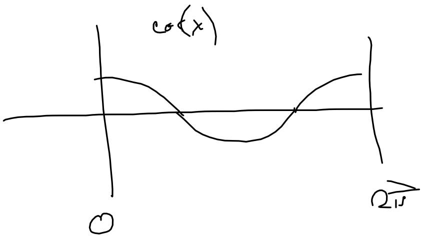
Wolfram Alpha - integrate, differentiate, graph and solve equations.

wolframalpha.com

Period of $\cos(\pi x)$:

want $\pi x = 2\pi$

$x = 2$

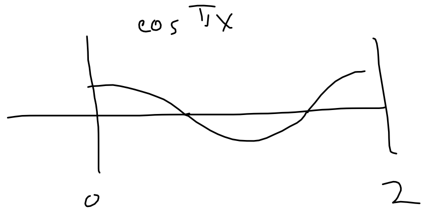


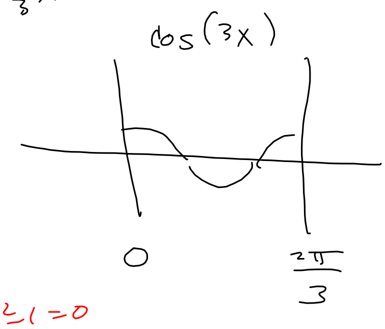
$\cos(\pi x)$

$x \mapsto \frac{1}{\pi} x$

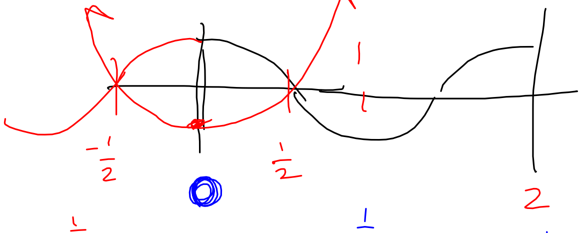
$\cos(3x)$

$x \to \frac{1}{3} x$





$\cos \pi x$ & $4x^2-1$



$4x^2 - 1 = 0$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} (\text{up} - \text{low}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} = 2\int_0^{\frac{1}{2}} \left(\cos(\pi x) - 4x^2 + 1\right) dx$$

symmetry.

$u = \pi x$

$du = \pi\, dx$

FORGOT THE "2"

$$= \frac{1}{\pi}\int_0^{\frac{1}{2}} \cos(\pi x)\,\pi\, dx - \int_0^{\frac{1}{2}} 4x^2 dx + \int_0^{\frac{1}{2}} 1\, dx$$

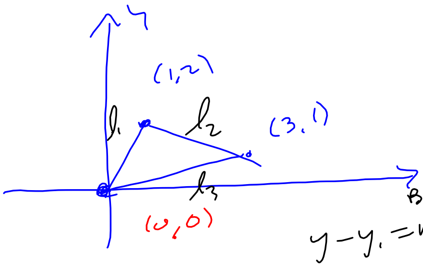
$$= \frac{1}{\pi}\left[\sin(\pi x)\right]_0^{\frac{1}{2}} - \frac{4}{3}x^3\Big]_0^{\frac{1}{2}} + x\Big]_0^{\frac{1}{2}}$$

$$= \frac{1}{\pi}\left[1 - 0\right] - \left[\frac{4}{3}\left(\frac{1}{8}\right) - \frac{4}{3}(0)\right] + \left[\frac{1}{2} - 0\right]$$

$$\frac{1}{\pi} - \frac{1}{6} + \frac{1}{2} = \frac{6 - \pi + 3\pi}{6} = \frac{1}{\pi} - \frac{1}{6} + \frac{3}{6} = \frac{1}{\pi} + \frac{1}{3}$$

TIMES 2: $\boxed{\frac{2}{\pi} + \frac{2}{3}}$

$(0,0), (3,1), (1,2)$



ME: $y = m(x - x_1) + y_1$

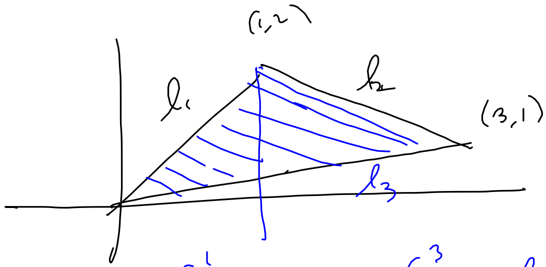
BOOK $y - y_1 = m(x - x_1)$

l_1: $m = \frac{2-0}{1-0} = 2 \quad y = 2(x-0)+0 = \boxed{2x = l_1(x)}$

l_2: $\frac{1-2}{3-1} = \frac{-1}{2} = m \quad y = \boxed{-\frac{1}{2}(x-1)+2 = l_2(x)}$

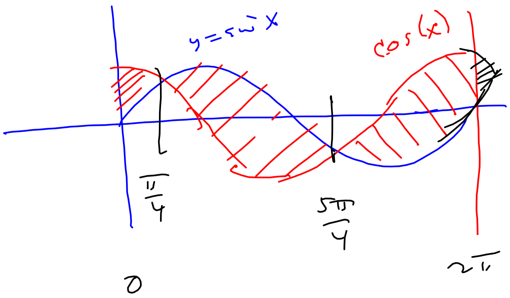
$= -\frac{1}{2}x + \frac{1}{2} + \frac{4}{2} = -\frac{1}{2}x + \frac{5}{2} = l_2(x)$

l_3: $m = \frac{1-0}{3-0} = \frac{1}{3} \quad y = \frac{1}{3}(x-0)+0 = \boxed{\frac{1}{3}x = l_3(x)}$



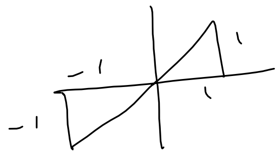
$$\int_0^1 l_1 - l_3 + \int_1^3 l_2 - l_3 = \int_0^1 \left(2x - \frac{1}{3}x\right) dx + \int_1^3 \left(-\frac{1}{2}x - \frac{1}{3}x\right) dx$$

Area bdd by sine & cosine from $x=0$ to $x=2\pi$



$\sin x = \cos x$

$\frac{\sin x}{\cos x} = \tan x = 1$



$\frac{\pi}{4}, \frac{5\pi}{4}$

$$\int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx$$

$$+ \int_{\frac{5\pi}{4}}^{2\pi} (\cos x - \sin x)\,dx$$

$$= \int_{\frac{5\pi}{4}}^{\frac{9\pi}{4}} (\cos x - \sin x)\,dx$$

$2\pi + \frac{\pi}{4} = \frac{8\pi + \pi}{4} = \frac{9\pi}{4}$

$y = \sqrt{2x+6}$

$y^2 = 2x+6$

$y^2 - 6 = 2x$

$\frac{1}{2}y^2 - 3 = \frac{y^2-6}{2} = x = g_1(y)$

$x = \frac{1}{2}y^2 - 3$

$(x-1)^2 = -\left(\sqrt{2x+6}\right)^2$

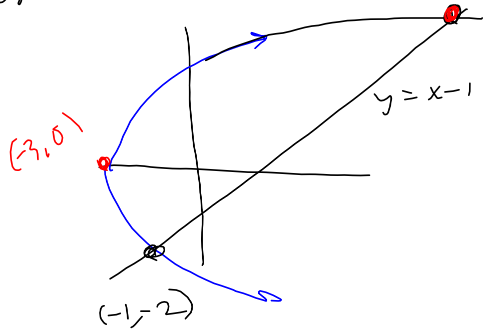
$(x-1) = -\sqrt{2x+6}$

$x^2 - 2x + 1 = 2x + 6$

$x^2 - 4x - 5 = (x-5)(x+1)$

$x - 1 = \sqrt{2x+6}$

$x = 5, -1$



$y = x - 1 \Longrightarrow x = y + 1$

$$\int_{-2}^{4} (\text{RIGHT} - \text{LEFT})$$

$$= \int_{-2}^{4} \left((y+1) - \left(\tfrac{1}{2}(y^2-6)\right)\right) dx$$

$$\int_{-3}^{-1} \left(\sqrt{2x+6} - \left(-\sqrt{2x+6}\right)\right) dx$$

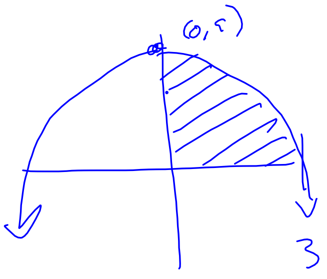
$$+ \int_{-1}^{5} \left(\sqrt{2x+6} - (x-1)\right) dx$$

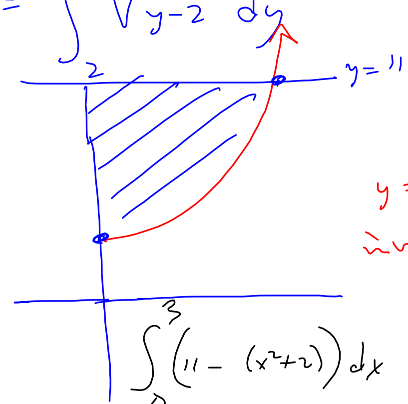
$$\int_0^3 (9-x^2)\,dx = \int_2^{11} \sqrt{y-2}\,dy$$

$x = \sqrt{y-2}$

$x^2 = y-2$

$y = x^2+2$





$y = \sqrt{x-2}$ has inverse function

$y = x^2+2$

$$\int_0^3 \left(11-(x^2+2)\right)dx = \int_0^3 (9-x^2)\,dx$$

$\sqrt{11-2} = \sqrt{9} = 3$

$(3, 11)$

$x^2+2 = 11$

$x^2 = 9$

$x = \pm 3$

$(\pm 3, 11)$

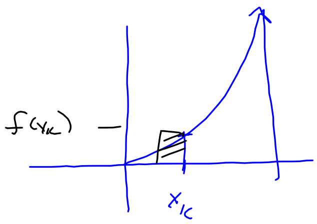
$$\int_1^4 7x^2\,dx = 7\int_1^4 x^2\,dx \qquad a=1,\ b=4$$

$$\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$$

$$x_k = a + k\Delta x = 1 + k\cdot\frac{3}{n}$$

$$= \frac{3k}{n} + 1 = \frac{3k+n}{n} \qquad \frac{3k}{n} + \frac{1}{1}\cdot\frac{n}{n} = \frac{3k+n}{n}$$

$$\sum_{k=1}^{n} f(x_k)\Delta x$$



$$= \Delta x \sum_{k=1}^{n} f(x_k)$$

$$\text{Area} = f(x_k)\Delta x$$

$$= \frac{3}{n}\sum 7x_k^2 = \frac{21}{n}\sum x_k^2 = \frac{21}{n}\sum\left(\frac{3k+n}{n}\right)^2$$

$$= \frac{21}{n}\sum \frac{9k^2+6kn+n^2}{n^2} = \frac{21}{n^3}\sum\left(9k^2+6kn+n^2\right)$$

$$= \frac{21}{n^3}\left[9\sum k^2 + 6n\sum k + n^2\sum 1\right]$$

$$= \frac{21}{n^3}\left[9\left[\frac{n^3+\cdots}{3}\right] + 6n\left[\frac{n^2+\cdots}{2}\right] + n^2[n]\right]$$

$$= \frac{63}{n^3}\left[n^3+\cdots\right] + \frac{21}{n^3}\left[3n(n^2+\cdots)\right] + \frac{21}{n^3}\left[n^3\right]$$

$$= \frac{63}{n^3}\left[n^3+\cdots\right] + \frac{63n^3+\cdots}{n^3} + 21 \xrightarrow{n\to\infty}$$

$$= 63 + 63 + 21 = 126 + 21 = 147$$

$$\int_0^{\frac{\pi}{3}} \sec^5 x \sin x \, dx =$$

$$= \int_0^{\frac{\pi}{3}} \sec^4 x \cdot \frac{1}{\cos x} \cdot \sin x \, dx$$

$$= \int_0^{\frac{\pi}{3}} \sec^4 x \tan x \, dx$$

$$= \int_0^{\frac{\pi}{3}} \sec^3 x \sec x \tan x \, dx$$

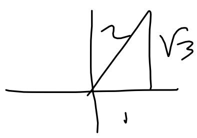
$u = \sec x \Rightarrow$

$du = \sec x \tan x \, dx$

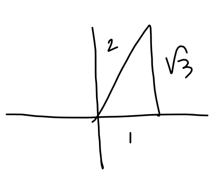
$$= \int_{x=0}^{x=\frac{\pi}{3}} u^3 \, du = \frac{1}{4} u^4 \Big]_{x=0}^{x=\frac{\pi}{3}} = \frac{1}{4} \sec^4 x \Big]_0^{\frac{\pi}{3}}$$

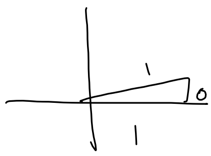
$$= \frac{1}{4} \left[\sec^4\left(\frac{\pi}{3}\right) - \sec^4(0) \right]$$

$\sec\left(\frac{\pi}{3}\right) = 2$



$$= \frac{1}{4} \left[\frac{1}{\cos^4\left(\frac{\pi}{3}\right)} - \frac{1}{\cos^4(0)} \right] -$$





$$= \frac{1}{4} \left[\frac{1}{\left(\frac{1}{2}\right)^4} - \frac{1}{1} \right] =$$

$$= \frac{1}{4} \left[2^4 - 1 \right] = \frac{1}{4} \left[16 - 1 \right] = \frac{15}{4}$$