$$\lim_{x\to 4} \frac{x^3-64}{x^2-16} = \lim f(x)$$

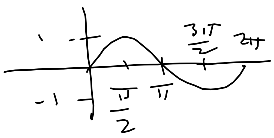
$$a^3-b^3 = (a-b)(a^2+ab+b^2)$$
$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$\frac{\cancel{(x-4)}(x^2+4x+4^2)}{\cancel{(x-4)}(x+4)} \xrightarrow{x\to 4} \frac{16+16+16}{8} = 6 = \lim f(x)$$

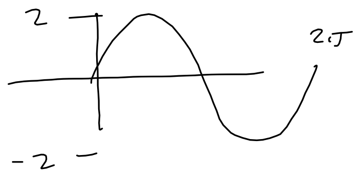
Fall '17 #3

$$\int_0^{\frac{\pi}{2}} |2\sin x - 1|\, dx$$

sin x

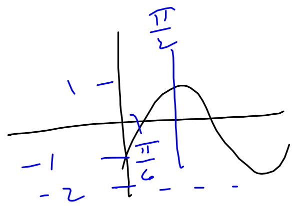


2 sin x



$$|f(x)| = \begin{cases} f(x) & \text{if } f(x) \ge 0 \\ -f(x) & \text{if } f(x) < 0 \end{cases}$$

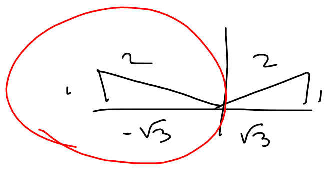
$$|x| = \begin{cases} x, & \text{if } x \ge 0 \\ -x, & \text{if } x < 0 \end{cases}$$



$$\sqrt{x^2} = |x|$$

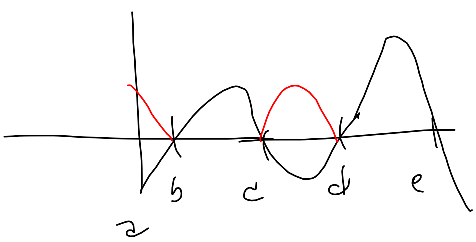
$$2\sin x - 1 = 0$$
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$

$$\int_0^{\frac{\pi}{2}} f = \int_0^{\pi/6} f + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} f$$



$$-\int_0^{\frac{\pi}{6}} \sin x\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sin x\, dx$$

$$2^2 - 1^2 = 4 - 1 = 3 = b^2$$
$$b = \sqrt{3}$$



$$\int_0^e |f(x)|\, dx = -\int_a^b f + \int_b^c f - \int_c^d f + \int_d^e f$$

$$\int (3x+1)^4\,dx = \frac{1}{3}\int (3x+1)^4(3\,dx) = \frac{1}{3}\int u^4\,du = \frac{1}{3}\left(\frac{(3x+1)^5}{5}\right)+C$$

$u=3x+1$

$$= \int u^4 \frac{du}{3} = \frac{1}{3}\int u^4\,du = \frac{1}{3}\cdot\frac{u^5}{5}+C =$$

$\Rightarrow du = 3\,dx$

$$\boxed{dx = \frac{du}{3}}$$

$-3x < 5$

$$\frac{}{-3} \;>\; \frac{}{-3}$$

$3x = 5$

$$\frac{-3x}{-3} < \frac{5}{-3}$$

$x > -\frac{5}{3}$

$x = \frac{5}{3}$

$$\int (3x+1)^4 x^2\,dx$$

$u = 3x+1 \Rightarrow u-1 = 3x \Rightarrow x = \frac{u-1}{3}$

$du = 3dx \Rightarrow dx = \frac{du}{3}$

$$\int (u)^4\left(\frac{u-1}{3}\right)^2\frac{du}{3} = \int u^4\left(\frac{u^2-2u+1}{9}\right)\frac{du}{3}$$

$(u-1)^2 = u^2-2u+1$

$$= \frac{1}{27}\int u^4(u^2-2u+1)\,du$$

$$= \frac{1}{27}\int (u^6-2u^5+u^4)\,du = \frac{1}{27}\left[\frac{u^7}{7}-\frac{2}{6}u^6+\frac{1}{5}u^5\right]+C$$

$\frac{1}{3}$

$$= \frac{1}{27(7)}u^7 - \frac{1}{3\cdot 27}u^6 + \frac{1}{27\cdot 5}u^5 + C$$

$$\boxed{= \frac{1}{7\cdot 27}(3x+1)^7 - \frac{1}{81}(3x+1)^6 + \frac{1}{135}(3x+1)^5 + C}$$

$$\int \sec^4 x \tan x\,dx$$

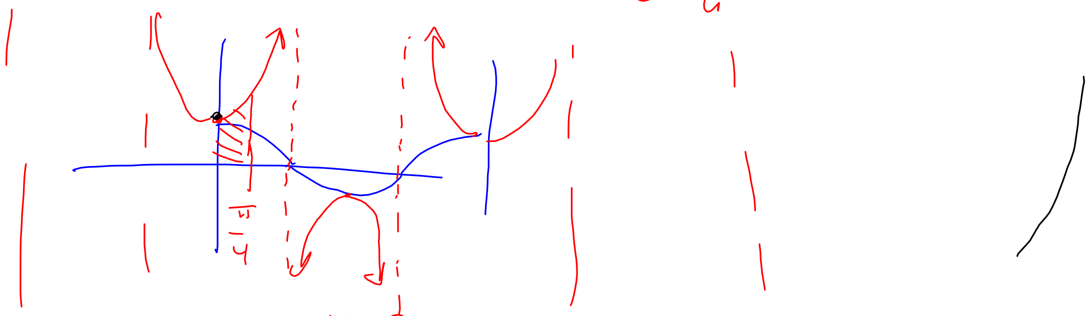
$$\frac{d}{dx}\left[\sec x\right] = \sec x \tan x$$

$u = \sec x \Rightarrow du = \sec x \tan x\,dx$

$$= \int_0^{\frac{\pi}{4}} \underbrace{(\sec^3 x)}_{u^3} \underbrace{(\sec x \tan x)\, dx}_{du} = \left[\frac{1}{4}\sec^4 x\right]_0^{\frac{\pi}{4}}$$

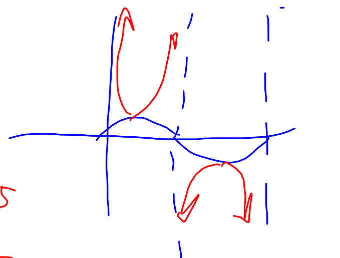
$$= \int u^3\, du = \frac{1}{4}u^4 + C$$

$$\int \underbrace{\sin^{-5} x}_{u^5} \underbrace{\cos x\, dx}_{du}$$



Not cont's @ x=0

$$\int_0^{\frac{\pi}{4}} \csc^3 x \cot x\, dx$$



$x=\pi$ $x=2\pi$

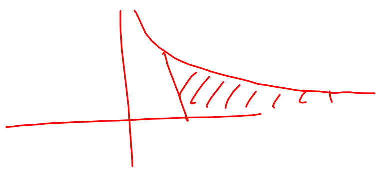
$$\int_0^{10} \frac{x^2+5}{x-5}\, dx$$

Blows up at $x=5$

FTC II DNA.

$$\int_1^{\infty} \frac{dx}{x} \quad \nexists$$

$$\int_1^{\infty} \frac{dx}{x^2} \quad \exists!$$



$$\int_0^{1} \frac{dx}{x^2} \quad \nexists$$

$$\int_0^{1} \frac{dx}{\sqrt{x}} \quad \exists!$$

$$\frac{d}{dx}\left[\int_0^x \frac{\cos(3t)}{t^2+4}\,dt\right] = \boxed{\frac{\cos(3x)}{x^2+4}}$$

$$\int_{x^2}^{\cos x} f(t)\,dt = \int_{x^2}^{0} f(t)\,dt + \int_0^{\cos x} f(t)\,dt$$

$$\int_a^b = \int_a^c + \int_c^b \qquad = -\int_0^{x^2} f(t)\,dt + \int_0^{\cos x} f(t)\,dt$$

$$\Rightarrow \frac{d}{dx}\int_{x^2}^{\cos x} f(t)\,dt = \frac{d}{dx}\left[-\int_0^{x^2} f(t)\,dt + \int_0^{\cos x} f(t)\,dt\right]$$

$$= -\frac{d}{dx}\int_0^{x^2} f(t)\,dt + \frac{d}{dx}\int_0^{\cos x} f(t)\,dt$$

$$= -f(x^2)\cdot 2x + f(\cos x)(-\sin x)$$

$$= \left(-\frac{\cos(3x^2)}{(x^2)^2+4}\right)(2x) + \left(\frac{\cos(3\cos x)}{(\cos(x))^2+4}\right)(-\sin x)$$