$$\int_1^7 (2x^2-3x)\,dx \qquad [a,b]=[1,7]$$

$$\Delta x = \frac{b-a}{n} = \frac{7-1}{n} = \frac{6}{n}$$

$$x_1 = a+\Delta x = 1+\frac{6}{n}$$

$$\vdots$$

$$x_k = 1+\frac{6k}{n}$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f\left(\frac{6k}{n}+1\right) = \frac{6}{n}\sum_{k=1}^{n}\left(2\left(\frac{6k}{n}+1\right)^2 - 3\left(\frac{6k}{n}+1\right)\right)$$

Yes

No

$$= \frac{6}{n}\cdot 2\sum_{k=1}^{n}\left(\frac{6k+1}{n}\right)^2 - \frac{6}{n}\cdot 3\sum_{k=1}^{n}\left(\frac{6k}{n}+1\right) = S_1 - S_2$$

$$\left(\frac{(6k+1)^2}{n^2} = \frac{(6k)^2+2(6k)(1)+1^2}{n^2} = \frac{36k^2+12k+1}{n^2}\right)$$

$$S_1 = \frac{12}{n}\sum_{k=1}^{n}\frac{36k^2+12k+1}{n^2} = \frac{12}{n^3}\left[\sum 36k^2 + \sum 12k + \sum 1\right]$$

$$= \frac{12}{n^3}\left[36\left(\frac{n^3+\cdots}{3}\right) + \cancel{12}^{\,6}\left(\frac{n^2+\cdots}{\cancel{2}}\right) + n\right]$$

$$= \frac{12\cdot 36}{3n^3}\left(n^3+\cdots\right) + \frac{12\cdot 6}{n^3}\left(n^2+\cdots\right) + \frac{12}{n^3}\cdot n$$

$$= \frac{4\cdot 36}{n^3}\cdot\left(n^3+\cdots\right) +$$

$$\sum k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+\cdots}{6} = \frac{n^3+\cdots}{3}$$

$$\sum_{k=1}^{n} f(x_k)\Delta x = \Delta x \sum_{k=1}^{n} f\left(\frac{6k}{n}+1\right) = \frac{6}{n}\sum_{k=1}^{n}\left(2\left(\frac{6k}{n}+1\right)^2 - 3\left(\frac{6k}{n}+1\right)\right)$$

$$\left(\frac{6k}{n}+1\right)^2 = \frac{36k^2}{n^2} + 2\left(\frac{6k}{n}\right)(1) + 1^2$$

Yes

$$= \frac{36k^2}{n^2} + \frac{12k}{n} + 1 \Longrightarrow$$

$$S_1 = \frac{6}{n}\sum_{k=1}^{n} 2\left(\frac{36k^2}{n^2} + \frac{12k}{n} + 1\right) = \frac{6}{n}\sum\left(\frac{72k^2}{n^2} + \frac{24k}{n} + 2\right)$$

$$\frac{6\cdot 72}{n^3}\sum k^2 + \frac{6}{n}\cdot\frac{24}{n}\sum k + \frac{6}{n}\cdot\sum_{k=1}^{n} 2$$

$$= \frac{6\cdot 72}{n^3}\left(\frac{n^3+\cdots}{3}\right) + \frac{6\cdot 24}{n^2}\left(\frac{n^2+\cdots}{2}\right) + \frac{6}{n}\cdot 2n$$

$$\xrightarrow{n\to\infty} 2\cdot 72 + 3\cdot 24 + 12$$

$$= 144 + 72 + 12 = 144 + 84 = \boxed{228 = S_1}$$

$$S_2 = \frac{6}{n}\cdot 3\sum_{k=1}^{n}\left(\frac{6k}{n}+1\right)$$

$$= \frac{18}{n}\cdot\frac{6}{n}\sum_{k=1}^{n}k + \frac{18}{n}\sum_{k=1}^{n}1$$

$$= \frac{6\cdot 18}{n^2}\left[\frac{n^2+\cdots}{2}\right] + \frac{18}{n}\cdot n$$

$$= \frac{3\cdot 18}{n^2}\left[n^2+\cdots\right] + 18 = \frac{54}{n^2}\cdot\left[n^2+\cdots\right] + 18$$

$$\xrightarrow{n\to\infty} 54+18 = \boxed{72 = S_2'}$$

Idiot check!

$$\int = S_1 - S_2 = 228-72 = 156 = \int_1^7 (3x^2-2x)\,dx$$

$$~~= \left[3\cdot\frac{x^3}{3} - 2\cdot\frac{x^2}{2}\right]_1^7 = \left[x^3-x^2\right]_1^7 = 7^3-7^2-(1^3-1^2)~~$$

$$~~= 7^2(7-1) = 7^2\cdot 6 = 294~~$$

$$\begin{array}{r} {}^{5}49 \\ 6 \\ \hline 294 \end{array}$$

$$\int_1^7 (2x^2-3x)\,dx = \left[\frac{2}{3}x^3 - \frac{3}{2}x^2\right]_1^7$$

$$= \left(\frac{2}{3}\left[7^3\right] - \frac{3}{2}\left[7^2\right]\right) - \left(\frac{2}{3}(1)^3 - \frac{3}{2}(1)^2\right)$$

$$\begin{array}{r} {}^{6}49 \\ 7 \\ \hline 343 \end{array}$$

$$= \frac{2\cdot 343}{3} - \frac{3\cdot 49}{2} - \left(\frac{2}{3}\cdot\frac{2}{2} - \frac{3}{2}\cdot\frac{3}{3}\right)$$

$$= \frac{4\cdot 343 - 9\cdot 49}{6} - \left(\frac{4-9}{6}\right)$$

$$\begin{array}{r} {}^{1}3{}^{1}43 \\ 4 \\ \hline 1372 \end{array} \qquad \begin{array}{r} {}^{8}49 \\ 9 \\ \hline 441 \end{array}$$

$$= \frac{1372-441}{6} - \left(\frac{-5}{6}\right)$$

$$= \frac{931+5}{6} = \frac{\cancel{936}^{\,468}}{\cancel{6}_{\,3}} = \boxed{156}$$

Let $f(x) = \int_0^x \frac{\sin(\pi t)}{t^2+1}\,dt \Rightarrow g(x) = f(2x^3)$

5. (10 pts) Find $g'(x)$, if $g(x) = \int_0^{2x^3} \frac{\sin(\pi t)}{t^2+1}\,dt$, i.e., evaluate $\frac{d}{dx}\left[\int_0^{2x^3} \frac{\sin(\pi t)}{t^2+1}\,dt\right]$.

$$\frac{d}{dx}\int_0^x \frac{\sin(\pi t)}{t^2+1}\,dt = \frac{\sin(\pi x)}{x^2+1}$$

$$\frac{d}{dx}\int_0^{2x^3} \frac{\sin(\pi t)}{t^2+1}\,dt = \left(\frac{\sin(2\pi x^3)}{(2x^3)^2+1}\right)(6x^2)$$

$$\frac{d}{dx}[\sin x] = \cos x$$

$$\frac{d}{dx}\left[\sin(x^3-7x)\right] = \left(\cos(x^3-7x)\right)(3x^2-7)$$

$(x-2)^5$

Binomial Theorem

$$\begin{array}{c} 1 \\ 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \end{array}$$

$$= x^5 + 5x^4(-2) + 10x^3(-2)^2 + 10x^2(-2)^3 + 5x(-2)^4 + (-2)^5$$

$$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$$

$$\left(\frac{5k}{n}+1\right)^3 = \left(\frac{5k}{n}\right)^3 + 3\left(\frac{5k}{n}\right)^2(1) + 3\left(\frac{5k}{n}\right)(1)^2 + 1^3$$

$$= \frac{125k^3}{n^3} + \frac{75k^2}{n^2} + \frac{15k}{n} + 1$$