Recall: Even f:

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

odd f:

$$\int_{-a}^{a} f(x)\,dx = 0$$

$$\int_{a}^{a} f(x)\,dx = 0$$

$$\int_{a}^{b} f(x)\,dx = -\int_{b}^{a} f(x)\,dx$$

$$\int (cf + dg) = c\int f + d\int g$$

This means "$\int$" is a "linear operator"

(Also need $\int_{a}^{b} 0\,dx = 0$.)

$$\int_0^{16} \frac{x}{\sqrt{4x+9}}\,dx = \frac{1}{4}\int_{x=0}^{x=10} \frac{x}{\sqrt{u}}\,4dx$$

$u = 4x+9 \Rightarrow du = 4dx$

So what about the x?

$4x+9 = u$

$4x = u-9$

$x = \frac{u-9}{4}$

$$= \frac{1}{4}\int_{x=0}^{x=10} \frac{\frac{u-9}{4}}{\sqrt{u}}\,du$$

2 ways of handling this: ① Change the limits to g(u) ② Leave alone & plug in after you un-substitute.

$$\frac{\frac{u-9}{4}}{\sqrt{u}} = \frac{u-9}{4\sqrt{u}}$$

$$= \left(\frac{u-9}{4}\right)u^{-\frac{1}{2}} = \frac{1}{4}\left[u^{\frac{1}{2}} - 9u^{-\frac{1}{2}}\right]$$

$$= \frac{1}{16}\int_{x=0}^{x=10}\left(u^{\frac{1}{2}} - 9u^{-\frac{1}{2}}\right)du = \frac{1}{16}\left[\frac{2}{3}u^{\frac{3}{2}} - 18u^{\frac{1}{2}}\right]_{x=0}^{x=10}$$

M1

$$= \frac{1}{16}\left[\frac{2}{3}(4x+9)^{\frac{3}{2}} - 18(4x+9)^{\frac{1}{2}}\right]_0^{10}$$

M2 $x=0 = \frac{u-9}{4} \Rightarrow u-9=0 \Rightarrow u=9$

$x=10 = \frac{u-9}{4} \Rightarrow u-9=40 \Rightarrow u=49$

$$\frac{1}{16}\left[\frac{2}{3}(u)^{\frac{3}{2}} - 18u^{\frac{1}{2}}\right]_9^{49}$$

Convert everything to the u-variable.

M1 $\frac{1}{16}\left[\frac{2}{3}(4(10)+9)^{\frac{3}{2}} - 18(4(10)+9)^{\frac{1}{2}} - \left(\frac{2}{3}(4(0)+9) - 18(4(0)+9)^{\frac{1}{2}}\right)\right]$

$$= \frac{1}{16}\left[\frac{2}{3}[49]^{\frac{3}{2}} - 18[49]^{\frac{1}{2}} - \left(\frac{2}{3}(9)^{\frac{3}{2}} - 18(9)^{\frac{1}{2}}\right)\right]$$

$$= \frac{1}{16}\left[\frac{2}{3}(343) - 18(7) - \left(\frac{2}{3}(27) - 18(3)\right)\right]$$

$$(49)^{\frac{3}{2}} = \left((49)^3\right)^{\frac{1}{2}} = \left((49)^{\frac{1}{2}}\right)^3 = 7^3 = 343 \quad (49)(7)$$

$$9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$$

$$= \frac{1}{16}\left[\frac{686}{3} - \frac{126}{1}\cdot\frac{3}{3} - \left(\frac{54}{3} - \frac{54}{1}\cdot\frac{3}{3}\right)\right]$$

$126 \times 3 = 378$, 54×3

$$= \frac{1}{16}\left[\frac{686 - 378 - 54 + 162}{3}\right] = \frac{1}{16}\left[\frac{848 - 432}{3}\right] = \frac{1}{16}\left[\frac{416}{3}\right] = \frac{26}{3}$$

$$\left(\frac{416}{16} = \frac{208}{8} = \frac{104}{4} = \frac{52}{2} = 26\right)$$

$$\int_0^{\frac{\pi}{2}} (2x+3)\sin(x^2+3x)\,dx = \left[-\cos(x^2+3x)\right]_0^{\frac{\pi}{2}}$$

($u = x^2+3x$, $du = (2x+3)\,dx$
About as brief as you can make it)

$$= -\cos\left(\left(\tfrac{\pi}{2}\right)^2 + 3\tfrac{\pi}{2}\right) - (-\cos(0))$$

$$= -\cos\left(\frac{\pi^2}{4} + \frac{3\pi}{2}\right) + 1$$

ouch!

Sum

$\cos(u+v) = \cos u\cos v - \sin u \sin v$

$\sin(u+v) = \sin u\cos v + \sin v\cos u$

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$ Power Reduction.

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

one path: Not very satisfactory

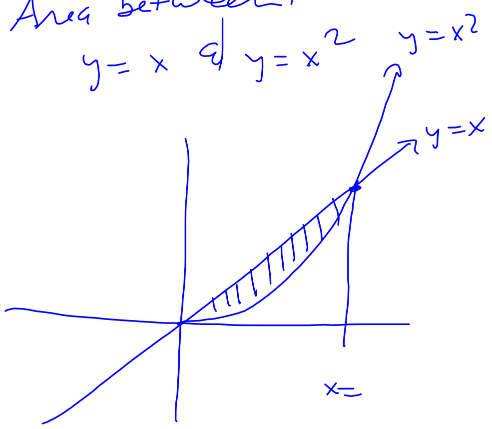
$$-\left[\cos\frac{\pi^2}{4}\cos\frac{3\pi}{2} - \sin\frac{\pi^2}{4}\sin\frac{3\pi}{4}\right] + 1$$

$$= -\left[0 - \sin\left(\frac{\pi^2}{4}\right)\left(-\frac{1}{\sqrt{2}}\right)\right] + 1$$

$= \frac{1}{\sqrt{2}}\sin\frac{\pi^2}{4} + 1$ is about as far as you can go, by hand, until Calculus II, when we learn Taylor's Series.

Area between:

$y = x$ & $y = x^2$

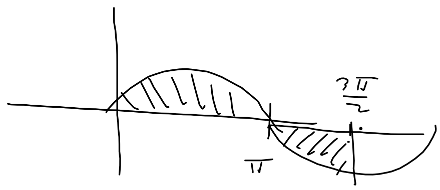


$$\int_0^1 (\text{upper} - \text{lower})$$

$$= \int_0^1 (x - x^2)\,dx$$

$$\int_a^b |f(x)|\,dx$$

$$\int_0^{\frac{3\pi}{2}} |\sin x|\,dx$$



$$= \int_0^{\pi} \sin x\,dx - \int_{\pi}^{\frac{3\pi}{2}} (\sin x)\,dx = -\cos x\Big]_0^{\pi} + \cos x\Big]_{\pi}^{\frac{3\pi}{2}}$$

$$= -(-1) - (-1) + 0 - (-1) = 3$$

$$= 3 \int_0^{\frac{\pi}{2}} \sin x\,dx \;?$$

$F(b) - F(a)$

$$= 3\left[-\cos x\right]_0^{\frac{\pi}{2}} = 3\left[0 - (-1)\right] = 3.$$