Give an upper & lower bound on

$$\int_a^b f(x)\,dx = \int_0^5 (3x^3-2x)\,dx$$

By EVT,

$\boxed{f(0)=0}$, $f(5) = 3(125)-2(5) = 375-10 = \boxed{365 = f(5)}$

$f'(x) = 9x^2-2 \overset{\text{SET}}{=} 0 \implies$

$9x^2 = 2$

$x^2 = \frac{2}{9}$

$x = \pm\sqrt{\frac{2}{9}} = \frac{\pm\sqrt{2}}{3}$

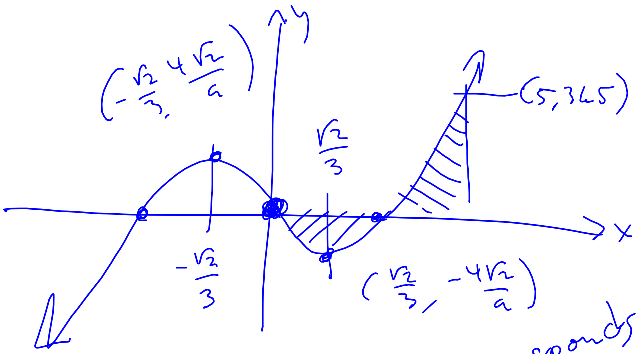
$f\left(\frac{\sqrt{2}}{3}\right) = 3\left(\frac{\sqrt{2}}{3}\right)^3 - 2\left(\frac{\sqrt{2}}{3}\right) = \frac{3(2\sqrt{2})}{3^3} - \frac{2\sqrt{2}}{3}$

$= \frac{2\sqrt{2}}{9} - \frac{2\sqrt{2}}{3}\cdot\frac{3}{3} = \frac{2\sqrt{2}-6\sqrt{2}}{9} = \frac{-4\sqrt{2}}{9} \approx -0.6285$

$f\left(-\frac{\sqrt{2}}{3}\right) = +\frac{4\sqrt{2}}{9}$ → $-\frac{\sqrt{2}}{3} \notin [0,5]$

This work says

$$\frac{-4\sqrt{2}}{9}(5) \le \int_0^5 (3x^3-2x)\,dx \le 365\cdot 5$$



$\int_0^5 f(x)\,dx$ corresponds to the shaded area.

$9x^2-2$

$3x^3-2x$

$x[3x^2-2]$

$x[\sqrt{3}x-\sqrt{2}][\sqrt{3}x+\sqrt{2}]$

$9x^2-2 = (3x)^2-(\sqrt{2})^2$

$= (3x-\sqrt{2})(3x+\sqrt{2})$

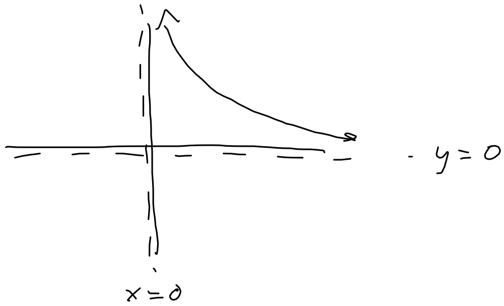
FTC Watchouts.

Differentiate:

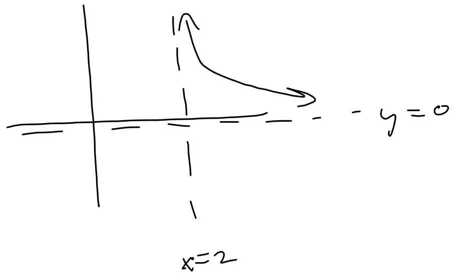
$\frac{d}{dx}\int_0^x \tan x \, dx$ only works for $0 \le x < \frac{\pi}{2}$

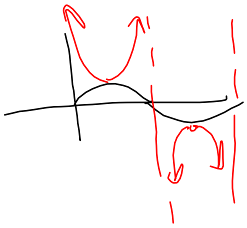
Evaluate: $\int_0^5 \frac{1}{\sqrt{x-2}}\,dx$ FTC II dne.

$\frac{1}{\sqrt{x}}$



$\frac{1}{\sqrt{x-2}}$





$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \csc(x)\,dx$ Bad.

$\csc(x)$ is not cont on $[\frac{\pi}{2}, \frac{3\pi}{2}]$

$\frac{d}{dx}$ & $\int dx$ are Linear Operators!

$$\frac{d}{dx}\left[af(x) + bg(x)\right] = \frac{d}{dx}\left[af(x)\right] + \frac{d}{dx}\left[bg(x)\right]$$
$$= a\frac{d}{dx}\left[f(x)\right] + b\frac{d}{dx}\left[g(x)\right]$$

$\forall a, b \in \mathbb{R}$, & $\forall f, g$ that are difbl.

Summarize

$$(af+bg)' = af' + bg'$$
$$\int (af+bg) = a\int f + b\int g$$

Flows from commutativity & associativity of addition & multiplication.

$$3(x^2+2x) = 3x^2+6x$$

$$3+2 = 2+3$$

$$\sum_{k=1}^{5} k = 1+2+3+4+5$$

$$\sum_{k=1}^{5}(k+k^2) = 1+1^2 + 2+2^2 + 3+3^2 + 4+4^2 + 5+5^2$$

$$= 1+2+3+4+5 + 1^2+2^2+3^2+4^2+5^2$$

$$= \sum_{k=1}^{5} k + \sum_{k=1}^{5} k^2$$

GOOD $\sum(a+b) = \sum a + \sum b$

BAD $\boxed{\sum(a+b)^2 \geq \sum a^2 + \sum b^2}$ WRONG!

$\sqrt{a+b} = \sqrt{a}+\sqrt{b}$ WRONG!

$(a+b)^2 = a^2+b^2$ WRONG!

Cauchy-Schwarz Inequality

$\sum(k\ \ )^2$ $\qquad$ $\sum k^2 + \sum k^2$

$(1+2)^2 + (3+4)^2$ $\qquad$ $1^2+3^2 + 2^2+4^2$

$= 9+49 = 54$ $\qquad$ $= 10 + 20 = 30$