$$\lim_{t\to\infty}\left(\sqrt{16t^2-17t}+4t\right)=\infty \quad (\nexists)$$

$$\lim_{t\to-\infty}\left(\sqrt{16t^2-17t}+4t\right) \, ?$$

$$\sqrt{16t^2-17t}+4t=\sqrt{16t^2\left(1-\frac{17t}{16t^2}\right)}+4t$$

$$=\sqrt{16t^2}\sqrt{1-\frac{17}{16t}}+4t$$

$\sqrt{16t^2}=4|t|=-4t$, since $t\to-\infty$
$\Rightarrow t<0 \Rightarrow |t|=-t$

$|x|=10 \Rightarrow x=\pm 10$

$$|t|=\begin{cases} t & \text{if } t\ge 0 \\ -t & \text{if } t<0\end{cases}$$

$$=-4t\sqrt{1-\frac{17}{16t}}+4t=-4t\left[\sqrt{1-\frac{17}{16t}}-1\right]$$

$$\xrightarrow{t\to-\infty}(+\infty)[0]$$

$(4t)^2=16t^2$

is indeterminate. That's why we do the $\sqrt{a}+b$

$$=\left(\frac{\sqrt{a}+b}{1}\right)\left(\frac{\sqrt{a}-b}{\sqrt{a}-b}\right)$$

$$=\frac{a-b}{\sqrt{a}-b}$$

In the future, we'll hospitalize the patient in these $\infty\cdot 0$, $\frac{0}{0}$, $\frac{\infty}{\infty}$, ∞^0, 0^∞ situations

$$\boxed{\text{L'Hôpital's Rule}}$$

$$\frac{\sin x}{x}\xrightarrow{x\to 0}1$$

$\frac{0}{0}$ sitch

$$\frac{\frac{d}{dx}[\text{top}]}{\frac{d}{dx}[\text{bottom}]}=\frac{\cos x}{1}\xrightarrow{x\to 0}1$$

TRY Rationalizing the numerator:

$$\left(\frac{\left(\sqrt{16t^2-17t}+4t\right)}{1}\right)\left(\frac{\sqrt{16t^2-17t}-4t}{\sqrt{16t^2-17t}-4t}\right)$$

$$=\frac{16t^2-17t-16t^2}{\sqrt{16t^2-17t}-4t}$$

$\sqrt{16t^2}=|4t|=4|t|$

$=-4t$ if $t<0$

$$=\frac{-17t}{-4t\sqrt{1-\frac{17}{16t}}-4t}$$

$$=\frac{-17t}{-4t\left(\sqrt{1-\frac{17}{16t}}+1\right)}=\frac{17}{4\left(\sqrt{1-\frac{17}{16t}}+1\right)}$$

$$\xrightarrow{t\to-\infty}\frac{17}{4(\sqrt{1}+1)}=\frac{17}{4(2)}\boxed{=\frac{17}{8}}$$

$f'(x) = \sec^2 x - \sin x$

$\Rightarrow \boxed{f(x) = \tan x - (-\cos x) + C}$

& $f(0) = 3 \Rightarrow$

$\tan(0) + \cos(0) + C = 3$

$\Rightarrow 1 + C = 3$

$\Rightarrow \boxed{C = 2}$

$y=3x+1$ to $(4,3)$

Let (x,y) on the graph of $y=3x+1$

Then distance to $(4,3)$ is

$$\sqrt{(x-4)^2+(y-3)^2} = \sqrt{(x-4)^2+(3x+1-3)^2}$$

$$= \sqrt{(x-4)^2+(3x-2)^2} = d$$

d to minimize d

we minimize $d^2 = D$

$$= x^2-8x+16 + 9x^2-12x+4$$

$$= 10x^2-20x+20 \Rightarrow$$

$$\frac{dD}{dx} = D' = 20x-20 \overset{\text{SET}}{=} 0$$

$$\Rightarrow 20x = 20$$

$$x = \frac{20}{20} = 1 \rightsquigarrow y = 3(1)+1$$

$$= 4$$

$$\Rightarrow \boxed{(x,y) = (1,4)}$$