MVT

f cnt^s on $[a,b]$

f difbl on (a,b)

$$\Rightarrow \exists\ c \in (a,b) \ni f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$\left(f'(c)(b-a) = f(b)-f(a) \right)$$

$f(x) = x + \cos(x)$ on $[0, 2\pi]$

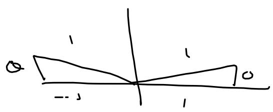
$f(0) = \cos(0) = 1$

$f(2\pi) = 2\pi + \cos(2\pi) = 2\pi + 1$

$$\Rightarrow \frac{f(b)-f(a)}{b-a} = \frac{2\pi+1-1}{2\pi-0} = \frac{2\pi}{2\pi} = 1$$

$f'(x) = 1 - \sin x \overset{SET}{=} 1$

$\Rightarrow \sin x = 0$



$x = 0, \pi, 2\pi$

We want $c \in (0, 2\pi)$

$\Rightarrow \boxed{c = \pi}$

$f(\pi) = \pi + \cos \pi$

$= \pi - 1$

EXTRA

Tangent Line @ $c = \pi$

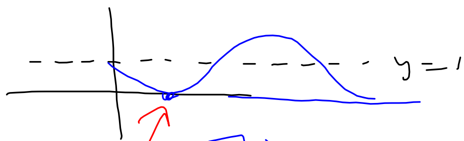
$y = L(x) = 1(x - (\pi)) + \pi - 1$

$= x - \pi + \pi - 1$

$L(x) = x - 1$

$\cos x + x$

$f'(x) = 1 - \sin x \overset{SET}{=} 0 \Rightarrow \sin x = 1$



This says no max/min for $f(x)$

can'tidate for extreme.

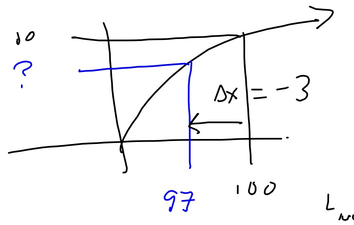
+ | +

$\frac{\pi}{2}$

Tangent line to approx. $\sqrt{97}$

$$f(x) = \sqrt{x} = x^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$



$$(x_1, y_1) = (100, f(100))$$

$$= (100, 10)$$

$$m = f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$$

$$L_{100}(x) = \frac{1}{20}(x-100)+10$$

$$L_{100}(97) = \frac{1}{20}(97-100) + 10$$

$$= \frac{-3}{20} + 10 = \frac{-3+200}{20} = \frac{197}{20}$$

$$= \frac{98.5}{10} = \boxed{9.85} \approx \sqrt{97}$$

Using Differential

$$f(x_2) \approx f(x_1) + f'(x_1)\Delta x$$

$$= f'(x_1)\Delta x + f(x_1)$$

$$= \left(\frac{1}{20}\right)(-3) + 10 = 9.85$$

MOAR DIFFERENTIALS

$$\Delta y = f(x_2) - f(x_1) \approx f'(x_1)\Delta x$$

$4\pi r^2$ $\qquad \frac{4\pi}{3}r^3$

πr^2 $\qquad 4\pi r^2$

$$\frac{d}{dx}\left[e^x\right] = e^x$$

Chain version

$$\frac{d}{dx}\left[e^{5x^2-7x}\right] = (10x-7)e^{5x^2-7x}$$

Solve: $e^{x-\frac{3}{2}} = 3x-2$

Find x-intercepts of

$e^{x-\frac{3}{2}} - 3x + 2$

$f(x) = g(x) \Longleftrightarrow$

$f(x) - g(x) = 0$