Find the unique $f(x)$ such that $f'(x) = x^2 - \cos(x)$

and $f(0) = 1$

Differential Equation.

$$f(x) = \int \left(x^2 - \cos(x)\right) dx$$

$$= \frac{x^3}{3} - \sin(x) + C$$

$$\left(\begin{array}{l} \frac{d}{dx}\left[\sin x\right] = \cos x \\ \int \cos x \, dx = \sin x + C \end{array} \right)$$

$$\Rightarrow f(0) = \frac{0^3}{3} - \sin(0) + C$$

$$= C = 1 \rightarrow$$

$$\boxed{f(x) = \frac{1}{3}x^3 - \sin(x) + 1}$$

$$f(x) = \frac{x^4 - x^3 - 8}{x^2 - x - 6} = \frac{\text{OWIE}}{(x-3)(x+2)} = \frac{(x-2)(x^3 + x^2 + 2x + 4)}{(x+2)(x-3)}$$

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & 0 & 0 & -8 \\ & & 2 & 2 & 4 & 8 \\ \hline & 1 & 1 & 2 & 4 & 0 \\ & x^3 & x^2 & x^1 & c & r \end{array}$$

$(x-2)(x^3 + x^2 + 2x + 4)$

& $x^3 + x^2 + 2x + 4$ has a real root

$\boxed{x \approx -1.477967242}$

wolframalpha.com

$\mathcal{D}: \mathbb{R} \setminus \{-2, 3\}$

$\boxed{\text{V.A.} \quad x = -2, x = 3}$

H.A. None

O.A.: Oblique (quadratic) asymptote

$$\frac{\text{Degree } 4}{\text{Degree } 2} \longrightarrow \text{Degree } (4-2) = 2$$

Done w/ Asymptote

Divisor

Dividend

$$
\begin{array}{r l}
 & x^2 + 6 \\
x^2 - x - 6 \,\big)\!\! & x^4 - x^3 + 0x^2 + 0x - 8 \\
 & -\left(x^4 - x^3 - 6x^2\right) \\
\hline
 & 6x^2 \quad \text{STOP!}
\end{array}
$$

$$\frac{x^4}{x^2} = x^2$$

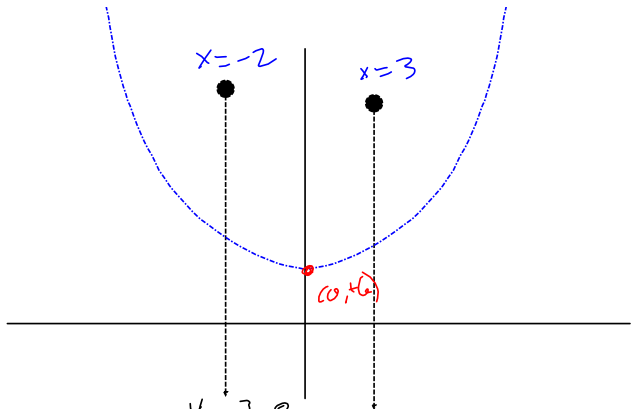
$$\frac{6x^2}{x^2} = 6$$

$$+ \frac{\text{small}}{x^2 - x - 6}$$

$$
\begin{array}{r l}
 & x^2 + 6 \;\; r\; 6x + 28 \\
x^2 - x - 6 \,\big)\!\! & x^4 - x^3 + 0x^2 + 0x - 8 \\
 & -\left(x^4 - x^3 - 6x^2\right) \\
\hline
 & 6x^2 + 0x - 8 \\
 & -\left(6x^2 - 6x - 36\right) \\
\hline
 & 6x + 28
\end{array}
$$

This says

$$f(x) = x^2 + 6 + \frac{6x + 28}{x^2 - x - 6}$$



$$f(x) = \frac{x^4 - x^3 - 8}{x^2 - x - 6} \Rightarrow$$

$$f'(x) = \frac{(4x^3 - 3x^2)(x^2 - x - 6) - (x^4 - x^3 - 8)(2x - 1)}{(x^2 - x - 6)^2}$$

$$(4x^3 - 3x^2)(x^2 - x - 6)$$

$$= 4x^5 - 4x^4 - 24x^3$$
$$-3x^4 + 3x^3 + 18x^2$$

$$4x^5 - 7x^4 - 21x^3 + 18x^2$$

$$(2x - 1)(x^4 - x^3 - 8)$$

$$= 2x^5 - 2x^4 + 0x^3 + 0x^2 - 16x$$
$$- x^4 + x^3 + 8$$

$$2x^5 - 3x^4 + x^3 - 16x + 8$$

$$4x^5 - 7x^4 - 21x^3 + 18x^2 -$$

$$= \frac{2x^5+3x^4-x^3+16x-8}{(\quad)^2}$$

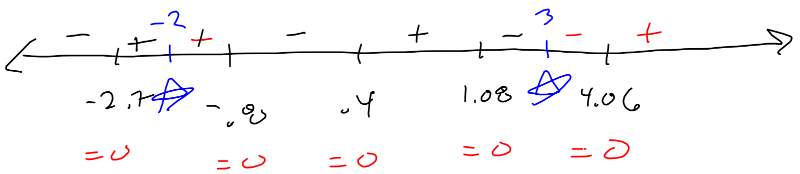
$$= \frac{2x^5-4x^4-22x^3+18x^2+16x-8}{(\quad)^2}$$

$$= \frac{2(x^5-2x^4-11x^3+9x^2+8x-4)}{(x^2-x-6)^2}$$

SET $= 0 \Rightarrow x =$

0.4110790456, 1.082024449, 4.058901063, -0.8071908715, -2.744813686

$(x-.4\ldots)^1(x-1.08\ldots)^1(x-4.059\ldots)^1(x+.807\ldots)^1(x+2.74\ldots)^1$



− +

Disagrees with parabolic end behavior.

How? Because this is f' and you're using end behavior of f(x), like an idiot, for ONE THING

Ignored the denominator

$(x^2-x-6)^2$

$(x+2)^2(x-3)^2$ −

− + No sign changes. +