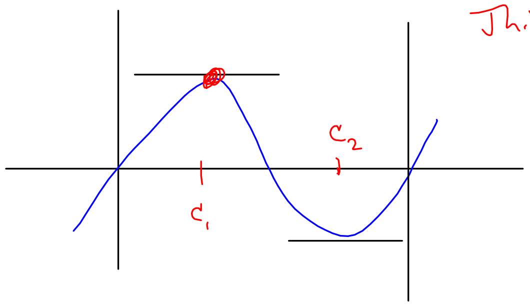
3.2

Rolle's

f cnt^s on $[a,b]$ &

f difb^l on (a,b) & $f(a)=f(b) \Longrightarrow$

$\exists\, c \in (a,b) \ni f'(c)=0$



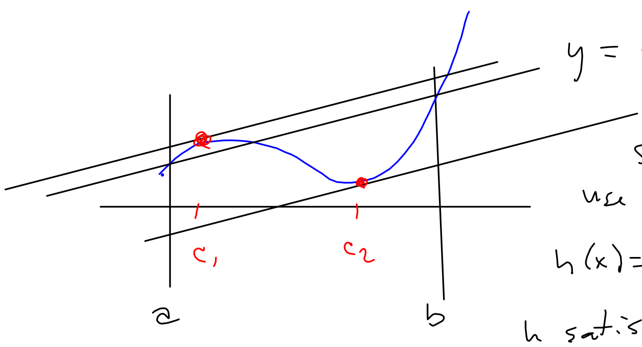
This one has 2!

Mean Value Theorem

f cnt^s on $[a,b]$

f difb^l on (a,b) $\Longrightarrow$

$$\exists\, c \in (a,b) \ni f'(c) = \frac{f(b)-f(a)}{b-a}$$



$$y = f(a) + \left(\frac{f(b)-f(a)}{b-a}\right)(x-a)$$

sketch of proof:

use Rolle's Theorem on

$$h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) + f(a)$$

h satisfies Rolle's which gives us MVT.

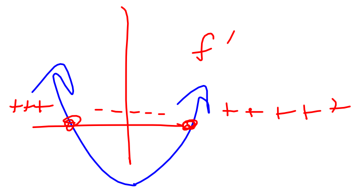
$f(x) = x^3 - 14x + c$ has at most one real root in $[-2, 2]$

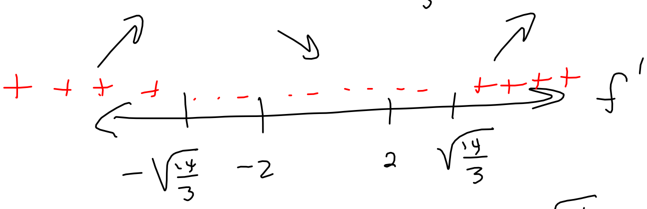
$\Rightarrow f'(x) = 3x^2 - 14$

$3x^2 = 14$

$x^2 = \frac{14}{3}$

$x = \pm\sqrt{\frac{14}{3}}$



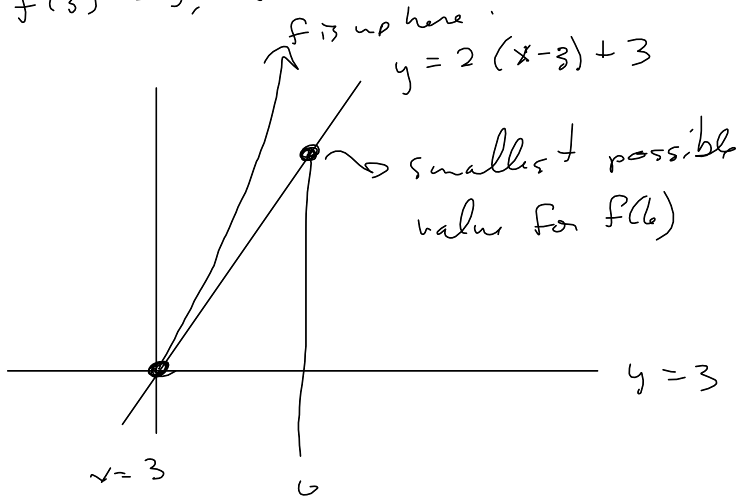


$f' < 0$ on $\left(-\sqrt{\frac{14}{3}}, +\sqrt{\frac{14}{3}}\right) \supsetneq [-2, 2]$

So it can only cross once

#25 Book #8 video

$f(3) = 3, \quad f'(x) \geq 2$!



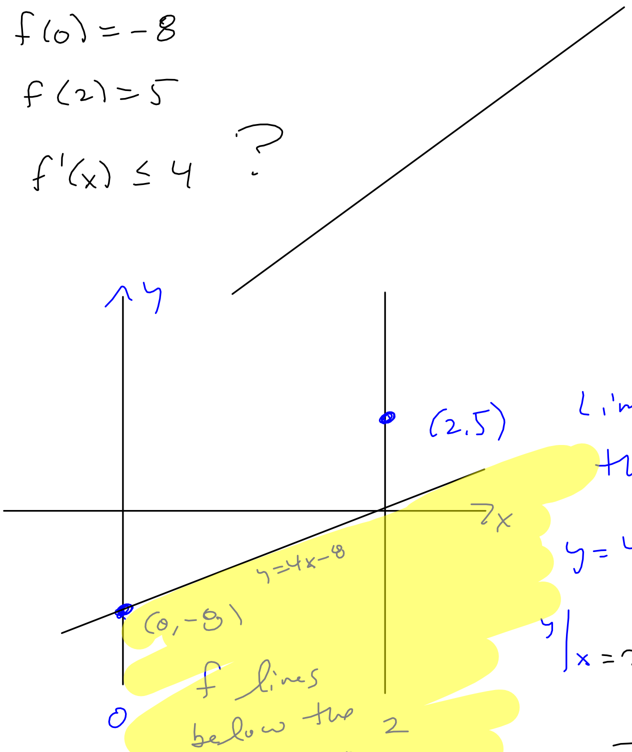
$f(6) \geq 2(6-3)+3 = 2(3)+3 = \boxed{9 \leq f(6)}$

This is related to what is called the "race-track principle."

$f(0) = -8$

$f(2) = 5$

& $f'(x) \le 4$?



Line w/ slope $m=4$ thru $(0,-8)$ is

$y = 4x - 8$

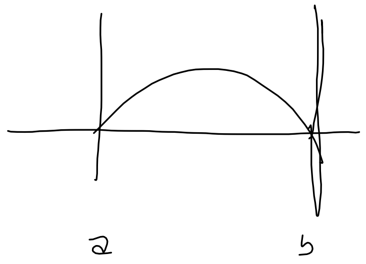
$y\big|_{x=2} = 4(2) - 8 = 0$

for $x \ge 0$: f lies below the line $y = 4x - 8$, so $f(2) \le 0 < 5 \Rightarrow$ impossible.

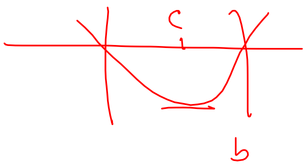
$\boxed{\text{No!}}$

Pf of Rolle's

S $f(x) > f(a)$ somewhere in (a,b).



By EVT, $\exists$ absolute max in (a,b).
By Fermat, that max, say, $(c, f(c))$ satisfies
$f'(c) = 0$ $\blacksquare$
(b/c $f'(x)$ exists on (a,b) & $f(c)$ is a max.
Same proof for this pic. Just a min:



Proof of MVT

Define $h(x) = f(x) - \frac{f(b)-f(a)}{b-a}(x-a) - f(a)$

Then $h(a) = 0$
$h(b) = f(b) - \frac{f(b)-f(a)}{b-a}(b-a) - f(a)$

$= f(b) - f(b) + f(a) - f(a) = 0 = h(a)$

So $h(c)$ satisfies Rolle's, so $\exists$ $c \in (a,b)$

$\Rightarrow h'(c) = 0$

But $h'(x) = f'(c) - \frac{f(b)-f(a)}{b-a} \cdot 1 - 0$

$= f'(c) - \frac{f(b)-f(a)}{b-a} = 0 \Rightarrow$

$\Rightarrow f'(c) = \frac{f(b)-f(a)}{b-a}$ $\blacksquare$