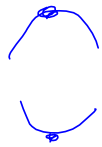
§3.1

D1 Let $c \in \mathcal{D}(f)$. Then $f(c)$ is an...

(a) absolute max if $f(c) \ge f(x) \quad \forall\, x \in \underline{\mathcal{D}(f)}$

(b) absolute min " $f(c) \le f(x) \quad \forall\, x \in \mathcal{D}(f)$

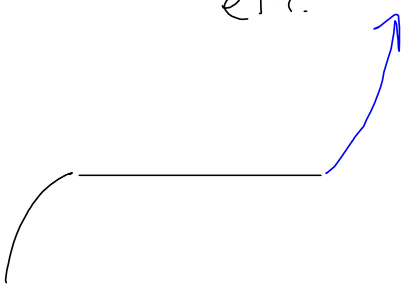


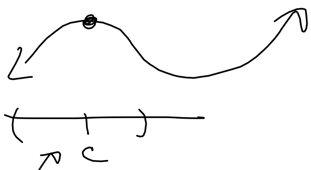
stuff.

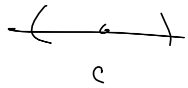
$\boxed{D2}$ $f(c)$ is a...

(a) local max if $f(c) \geq f(x)$ for x near c.

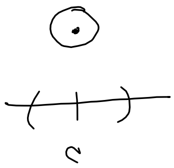
etc.







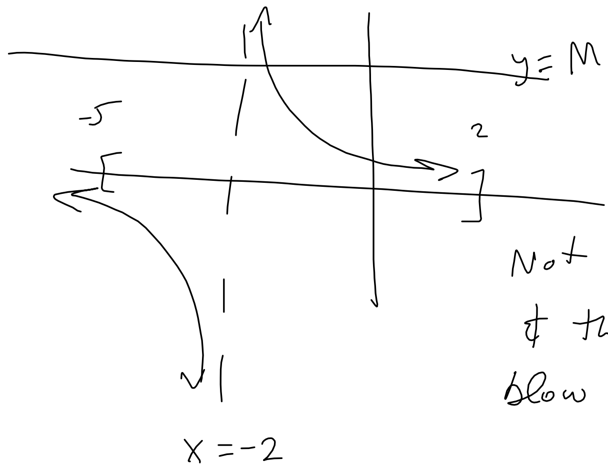
"Near"
Some open interval containing c.



EVT f is cnts on $[a,b]$ $\Longrightarrow$

f achieves its max and its min on $[a,b]$.

Nonexample $f(x) = \dfrac{1}{x+2}$ on $[-5, 2]$



I have a library of basic functions

Not cnts @ $x = -2 \in [-5, 2]$ & that allows it to blow up.

Prove there's no maximum.

Suppose it has one, say, $y = M$

Scratch: $\dfrac{1}{x+2} > M$ $\longrightarrow$ $1 > M(x+2)$

$\dfrac{1}{M} > x + 2$

$\boxed{\dfrac{1}{M} - 2 > x}$ → So if we're closer to 2 than $\frac{1}{M}$, $\frac{1}{x+2} > M$.

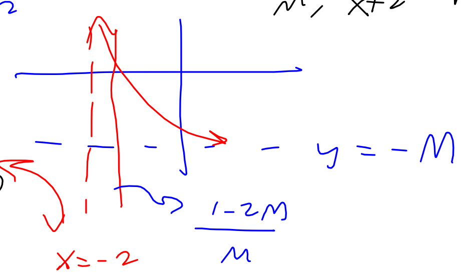
$\dfrac{1}{x+2} - M$

FORMAL

$\dfrac{1}{x+2} - M > 0$

$\dfrac{1 - M(x+2)}{x+2} > 0$

$\dfrac{1 - Mx - 2M}{x+2} = \dfrac{-Mx - 2M + 1}{x+2} > 0$

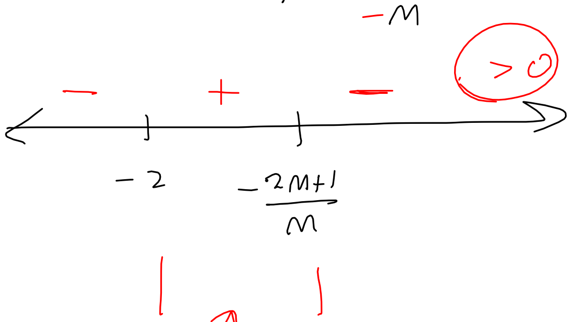


Critical #s:

$x = -2$ or $-mx - 2m + 1 = 0$

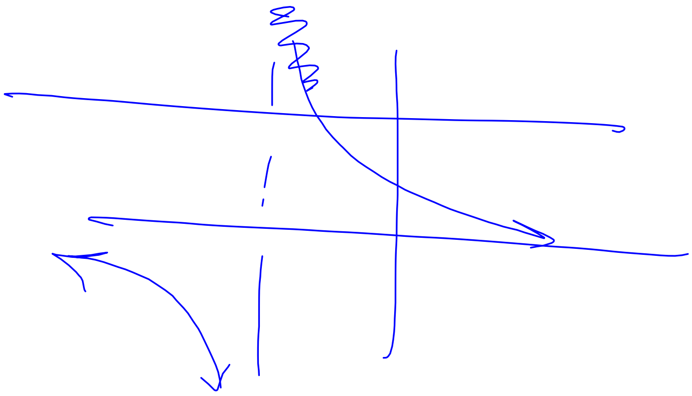
$-mx = 2m - 1$

$x = \frac{2m-1}{-m} = \frac{1-2m}{m} = \frac{1}{m} - 2$

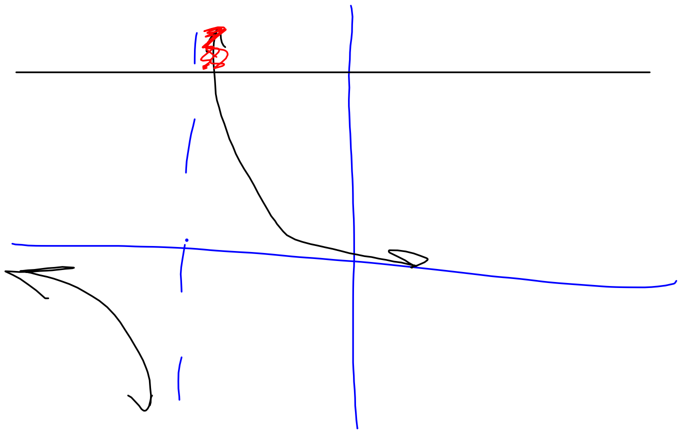


$\frac{1}{x+2} - m$

If we're in here, this thing is positive.

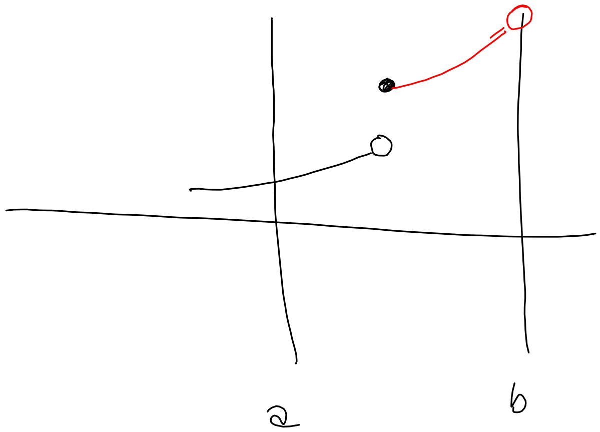


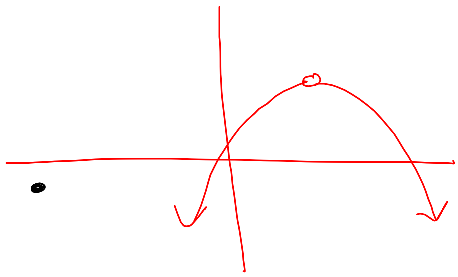
$x = -2$



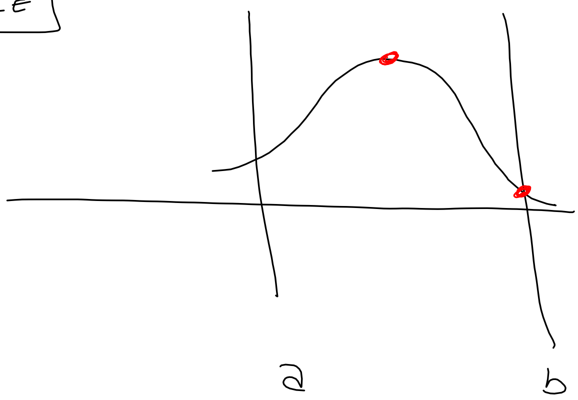
X = -2

So lack of continuity is a problem





EXAMPLE

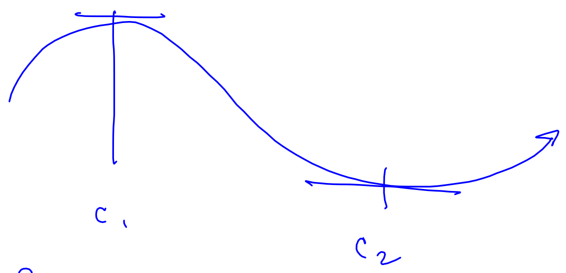


Closed interval method
Look for local extremes
AND check the
end points.

CALCULUS!

Critical #s $f(c)$ exists.

$f'(c) = 0$ OR $f'(c) \nexists$



$f(c_1)$ is max $\quad$ $f(c_2)$ is min

$f'(c_1) = 0$ $\quad$ $f'(c_2) = 0$

Fermat's Thm If $f'(c)$ exists and $f(c)$ is an extreme, then $f'(c) = 0$

If it rains, I bring an umbrella.

rain $\Longrightarrow$ umbrella.

The converse:

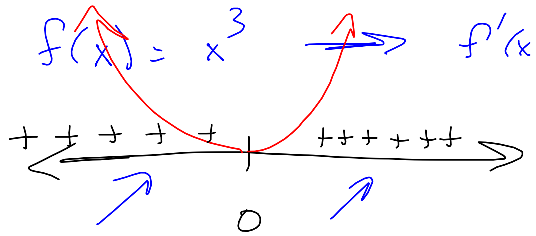
umbrella $\Longrightarrow$ rain.

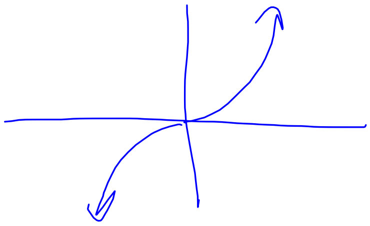
$f'(c) = 0 \Longrightarrow$ max/MIN.

The converse isn't equivalent to the original.

But still, $f'(c) = 0$ gives you something to work with.

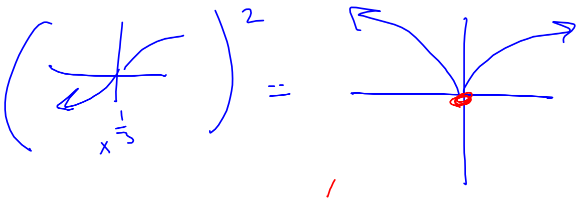
$f(x) = x^3 \Longrightarrow f'(x) = 3x^2 \Longrightarrow f'(x) = 0$ when $x = 0$





Now, <u>another</u> place where we have max/min is when $f'(x) \nexists$

$f(x) = x^{\frac{2}{3}} = \left(x^{\frac{1}{3}}\right)^2$



$f'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

$2/3\, x^{-1/3}$

Ambiguous

$f'(x) = \frac{2}{3} x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

$\nexists$ when $x = 0$

But $f(0) = 0$.