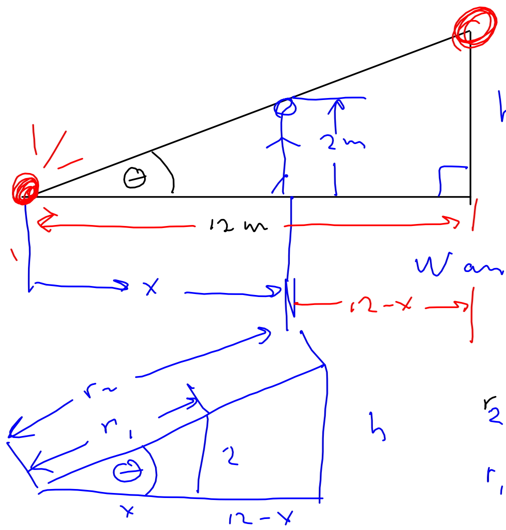
S 2.8 # 18

A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s how fast is the length of his shadow on the building decreasing when he is 4 m from the building?



Let t = time in seconds

h = height of shadow in m.

x = distance from spotlight in m.

Want: $\left.\frac{dh}{dt}\right|_{x=4}$

$r_2 = \sqrt{h^2 + 12^2}$

$r_1 = \sqrt{x^2 + 2^2}$

$\tan\theta = \frac{2}{x} = \frac{h}{12}$ By Similar Triangles.

We Know: $\frac{dx}{dt} = 1.6$ m/s

$$\frac{d}{dt}\left[\ 2x^{-1} = \frac{1}{12}x\ \right]$$

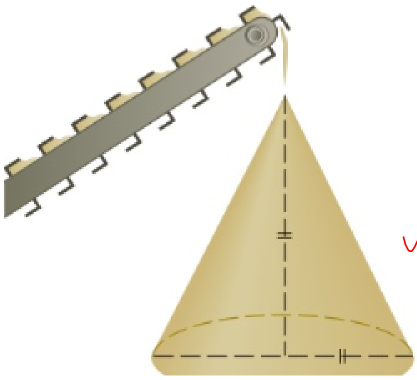
$$-x^{-2}\frac{dx}{dt} = \frac{1}{12}\cdot\frac{dh}{dt}$$

$$\left(-(4)^{-2}\right)\left(\frac{8}{5}\right) = \frac{1}{12}\frac{dh}{dt}$$

$$\Rightarrow -12\left(\frac{1}{\cancel{16}_{2}}\right)\left(\frac{\cancel{8}}{5}\right) = \frac{dh}{dt}$$

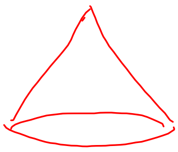
$$\boxed{\frac{dh}{dt} = -\frac{6}{5}\ \text{m/s} = -1.2\ \text{m/s}}$$

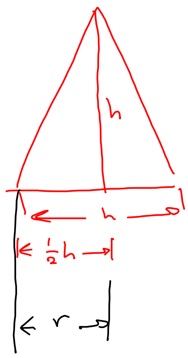
29. Gravel is being dumped from a conveyor belt at a rate of 30 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?



V = Volume of cone in ft^3.
t = time in seconds
h = height of cone in ft.
r = radius of the base in ft.

want: $\left.\frac{dh}{dt}\right|_{h=10}$





Given:

$\frac{dV}{dt} = 30\ \frac{ft^3}{min}$ & Diameter = height $\Rightarrow r = \frac{1}{2}h$

$\boxed{V = \frac{1}{3}\pi r^2 h} = \frac{1}{3}\pi\left(\frac{1}{2}h\right)^2 h = \frac{1}{12}\pi h^3$

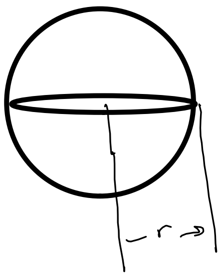
$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \frac{dh}{dt} = 30\ \frac{ft^3}{s}$

$\Rightarrow \frac{dh}{dt} = \frac{120}{\pi h^2}$

what's $\left.h\right|_{h=10} = 10!$ Duh.

$= \frac{120}{\pi(10)^2} = \frac{120}{100\pi} = \frac{6}{5\pi}\ ft/s \approx 0.3819718633\ ft/s.$

If a snowball melts so that its surface area decreases at a rate of 1 cm squared per minute, find the rate at which the diameter decreases when the diameter is 10 cm



S = surface area in cm^2

r = radius in cm

t = time, in minutes

Want $\left.\frac{dr}{dt}\right|_{r=5}$

Given:

$$\frac{dS}{dt} = -1 \ \frac{cm^2}{min} \qquad V = \frac{4}{3}\pi r^3$$

$$S = 4\pi r^2$$

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = -1$$

$$\Rightarrow (8\pi)(5)\frac{dr}{dt} = -1$$

$$\frac{dr}{dt} = \frac{-1}{40\pi} \ \frac{cm}{min}$$