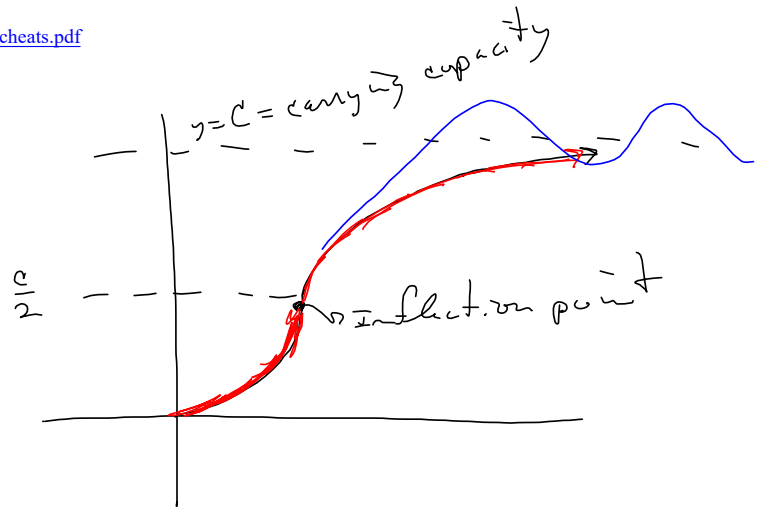


<https://harryzaims.com/201/201-fall-17/notes/170914-chapter-2-cheats.pdf>

$$P(t) = \frac{C}{1 + be^{-kt}}$$



§ 2.7 #20 $F = G \frac{m_1 m_2}{r^2} = G m_1 m_2 r^{-2}$

(a) $\Rightarrow \frac{dF}{dr} = -2 G m_1 m_2 r^{-3} = \frac{-2 G m_1 m_2}{r^3}$

(b) $\left. \frac{dF}{dr} \right|_{r=20,000 \text{ km}} = 2 \text{ N/km}$

Want $\left. \frac{dF}{dr} \right|_{10,000 \text{ km}}$

$$\frac{-2 G m_1 m_2}{(20,000)^3} = 2$$

$$-2 G m_1 m_2 = 2 (20,000)^3$$

$$\Rightarrow G = \frac{-20,000^3}{m_1 m_2}$$

$$\frac{20000^3}{10000^3} = \frac{2^3 (10000)^3}{10000^3} = 8$$

$$\frac{-2(-20000)^3 m_1 m_2}{(10,000)^3} = -2(-8) m_1 m_2 = \left. \frac{dF}{dr} \right|_{r=10000}$$

29. Suppose that the cost (in dollars) for a company to produce x pairs of a new line of jeans is

$$C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3$$

- Find the marginal cost function.
- Find $C'(100)$ and explain its meaning. What does it predict?
- Compare $C'(100)$ with the cost of manufacturing the 101st pair of jeans.

(29) Let x = the # of pairs of jeans produced &
 $C = C(x)$ = the cost of producing x pairs of jeans.

And $C(x) = .0002x^3 + .01x^2 + 3x + 2000$

Marginal cost is the cost of producing one more unit at given level of production x .

Formally: $\frac{C(x+1) - C(x)}{(x+1) - x} = C(x+1) - C(x)$

Practically:

Marginal cost = $C'(x)$!

$$C := x \mapsto .0002 \cdot x^3 + .01 \cdot x^2 + 3 \cdot x + 2000$$

$$C := x \mapsto 0.0002x^3 + 0.01x^2 + 3x + 2000$$

$$C_p := D(C)$$

$$C'(x) = C_p := x \mapsto 0.0006x^2 + 0.02x + 3$$

@ 100 units (pairs of jeans) C

$C(101) - C(100)$ = The cost of the 101st pair

$$11.0702$$

$C_p(100)$ = Marginal cost @ $x=100$ pairs produced.

$$11.0000$$

S'2.7 #21

$$A(x) = \frac{p(x)}{x}, \text{ where}$$

A = average production as a function of
 x = the # of workers, and
 $p(x)$ = production

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$\Rightarrow \textcircled{a} A'(x) = \frac{p'(x)x - p(x)}{x^2}$$

Why does the company want to add people if $A'(x) > 0$? Productivity is increasing. They're getting more in productivity by adding people. The benefit of adding workers is greater than the cost of adding them.

$$\textcircled{b} A'(x) > 0 \Rightarrow \frac{p'(x)x - p(x)}{x^2} > 0$$

$$\Rightarrow p'(x)x - p(x) > 0 \quad \text{b/c } x^2 > 0$$

$$\Rightarrow p'(x)x > p(x)$$

$$\Rightarrow p'(x) > \frac{p(x)}{x} \quad (\text{b/c } x > 0)$$

So we want $p'(x)$ greater than average production.

An aside: $\frac{B(x)}{(x-2)^3} > 0$

Does not imply $B(x) > 0$

$$(x-2)^3 \leftarrow \begin{array}{c} - \quad | \quad + \\ \hline 2 \end{array} \rightarrow$$

Suppose $B(x) = x^2 + 1$

$$\text{Then } \frac{x^2 + 1}{(x-2)^3} > 0$$

$$\leftarrow \begin{array}{c} - \quad | \quad + \\ \hline 2 \end{array} \rightarrow \frac{x^2 + 1}{(x-2)^3}$$

$$\text{So } \frac{x^2 + 1}{(x-2)^3} > 0 \text{ for } x > 2$$

$x^2 + 1 > 0$ is true but doesn't help.