a. $f(x)=x^{\frac{5}{2}}-3x^2+11x+5-2x^{-\frac{2}{3}}$

$\frac{5}{2}x^{\frac{3}{2}}-6x+11-\left(-\frac{2}{3}\right)\left(2x^{-5/3}\right)=f'(x)$

$-\frac{2}{3}-\frac{3}{3}$

$=-\frac{5}{3}$

b. $g(t)=\sin(5t)\cos(3t)$

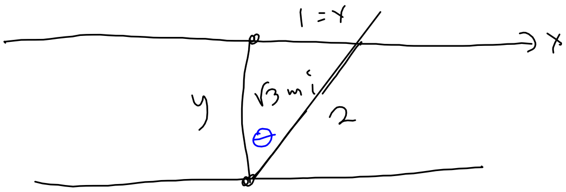
c. $h(\rho)=\dfrac{\tan(\rho)}{(2\rho-5)}$

d. $r(w)=(7w^2+5w)^6$

e. $Q(x)=\cos(6w)-6\cos(w)$ (It's a triiiiiick!)

$\to 6\left(7w^2+5w\right)^5(14w+5)=r'(w)$

5. (10 pts) A lighthouse is exactly $\sqrt{3}$ miles from the nearest point P on a straight shoreline, and its light makes 6 revolutions per minute. How fast is the beam of light moving along the shoreline, when it's 1 mile from P?



want $\left.\dfrac{dx}{dt}\right|_{x=1}$

$\left(\theta=\frac{\pi}{6}\right)$

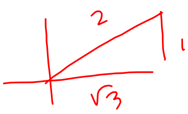
$$\left(\frac{6\text{ revs}}{\text{min.}}\right)\left(\frac{2\pi\text{ radians}}{1\text{ rev}}\right)=\frac{12\pi\text{ radians}}{\text{min}}$$

$$\frac{x}{y}=\tan\theta$$

$$x=y\tan\theta$$

$$\frac{dx}{dt}=\frac{dy}{dt}\tan\theta+y\left(\sec^2\theta\right)\left(\frac{d\theta}{dt}\right)$$

$$=0+\sqrt{3}\left(\sec^2\frac{\pi}{6}\right)\left(\frac{12\pi\text{ rad}}{\text{min}}\right)$$



$$=(\sqrt{3})\left(\frac{2}{\sqrt{3}}\right)^2(12\pi)=\frac{4}{\sqrt{3}}12\pi=\frac{48}{\sqrt{3}}\pi\frac{\text{mi}}{\text{min}}$$

$$=\left(\frac{48}{\sqrt{3}}\pi\right)\left(\frac{\text{mi}}{\text{min}}\right)\left(\frac{60\text{ min}}{1\text{ hr}}\right)\approx 5223.74\frac{\text{mi}}{\text{hr}}.$$

Filling a circular balloon @ a rate of $\frac{10 ft^3}{min}$. How fast is the radius growing when the volume is $50 ft^3$?

$$\frac{d}{dt}\left[V = \frac{4}{3}\pi r^3\right]$$

$$V = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{4}{3}\pi r^3 = 50$$

$$\left.\frac{dV}{dt}\right|_{V=50} = \left.4\pi r^2 \frac{dr}{dt}\right|_{V=50}$$

$$r^3 = \frac{3 \cdot 50}{4\pi}$$

$$r = \sqrt[3]{\frac{150}{4\pi}} = \sqrt[3]{\frac{75}{2\pi}} = r$$

$$= 4\pi\left[\sqrt[3]{\frac{75}{2\pi}}\right]^2 \frac{dr}{dt} = 10 \frac{ft^3}{min}$$

Solve for $\frac{dr}{dt}$ ..

Suppose the radius is growing at 1 cm/sec. How fast is the volume changing, when $V = 50$? cm^3

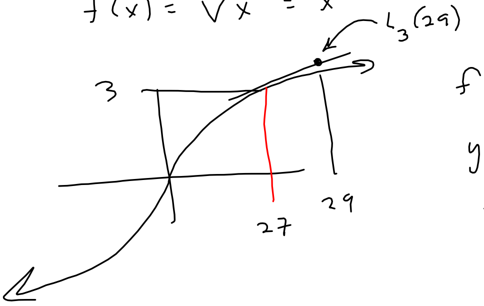
$$\ldots \quad \frac{dV}{dt} = 4\pi r^2 \left(\frac{1\,cm}{sec}\right)$$

$$= \boxed{4\pi\left[\sqrt[3]{\frac{75}{2\pi}}\right]^2 (1) \frac{cm^3}{sec}}$$

Differentials / Tangent Lines

Approximate $\sqrt[3]{29}$ using differentials (tangent line approx)

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$



$f'(x) = \frac{1}{3}x^{-\frac{2}{3}} \Rightarrow f'(27) = \frac{1}{3}(27)^{-2/3}$

$= \frac{1}{3}\left(\frac{1}{3}\right)^2 = \frac{1}{27} = m$

$y = m(x - x_1) + y_1$

$= \frac{1}{27}(x - 27) + 3$

$= f'(27)(x-27) + f(27)$

Plug in $x = 29$:

$L_{27}(29) = \frac{1}{27}(29-27) + 3$

$= \frac{1}{27}(2) + \frac{81}{27} = \frac{83}{27}$

```
27^(1/3)
                  3
29^(1/3)
        3.072316826
83/27
        3.074074074
```

When doing this with trig funcs, be sure to use radians.

$f(x) = \frac{(x-2)'(x+2)'}{(x-3)'} = \frac{x^2-4}{x-3}$ Improper $\frac{\deg 2}{\deg 1} \Rightarrow$ slant / oblique asymptote.

↳ Line.

$D = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$

V.A.: $x = 3$

O.A.: $y = x + 3$

$f(0) = \frac{0^2-4}{0-3} = \frac{4}{3}$

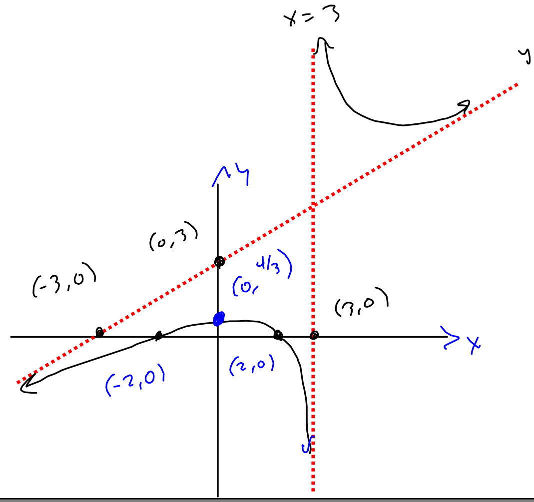
Divide:

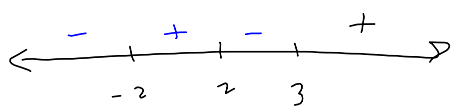
$\boxed{x+3 = y \text{ is O.A.}}$ r 5

$$\begin{array}{r} x+3 \\ x-3\,\overline{)\,x^2+0x-4} \\ -(x^2-3x) \\ \hline 3x-4 \\ -(3x-9) \\ \hline 5 \end{array}$$

$\frac{3x}{x} = 3$

$y = x+3 \quad f(x) = x+3 + \frac{5}{x-3}$



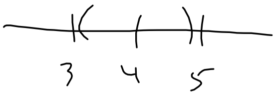


Claim: $x^2-3x+2 \xrightarrow{x \to 4} 6$

Scratch: $|x^2-3x+2-6| = |x^2-3x-4| = |x+1|\,|x-4| < |x+1|\,\delta$

Assume $\delta \le 1$. $3 < x < 5 \implies$

$4 < x+1 < 6$



$|x+1| < 6$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \min\{\frac{\varepsilon}{6}, 1\}$. Then if $0 < |x-4| < \delta$, we have

$$|x^2-3x+2-6| = |x^2-3x-4| = |x+1|\,|x-4| < 6\delta \le 6 \cdot \frac{\varepsilon}{6}$$

$$= \varepsilon \quad \blacksquare$$