$5x^2y^2 - 7xy^3 = x^2\sin(xy)$

Find $\frac{dy}{dx}$, by assuming y is a function of x

$10xy^2 + 10x^2yy' - 7y^3 - 21xy^2y' = (2x\sin(xy))(\cos(xy))[y + xy']$

$= 2xy\sin(xy)\cos(xy) + 2x^2\sin(xy)\cos(xy)y'$

$10x^2yy' - 21xy^2y' - 2x^2\sin(xy)\cos(xy)y' = -10xy^2 + 7y^3 + 2xy\sin(xy)\cos(xy)$

FACTOR out y'. Divide by coefficient of y'

$y'(10x^2y - 21xy^2 - 2x^2\sin(xy)\cos(xy)) =$ SAME RHS $\Rightarrow$

$$y' = \frac{-10xy^2 + 7y^2 + 2xy\sin(xy)\cos(xy)}{10x^2y - 21xy^2 - 2x^2\sin(xy)\cos(xy)}$$

Test 2 Fall '17

$f'g + fg'$

$x^2 - xy - y^2 = 1$

(a) y':

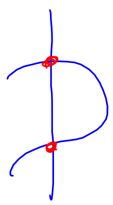
$2x - y - xy' - 2yy' = 0$

$-xy' - 2yy' = -2x + y$

$y' = \dfrac{-2x+y}{-x-2y}$

(b) Find $L(x)$ at $(2,1)$

Need b/c $x=2$ doesn't necessarily determine y uniquely.



$y'\Big|_{(2,1)} = \dfrac{-2(2)+1}{-2-2(1)} = \dfrac{-4+1}{-4} = \dfrac{3}{4}$

$\boxed{y = \frac{3}{4}(x-2) + 1}$

§2.4 $f(\theta) = \theta \sin\theta \implies \boxed{f'(\theta) = \sin\theta + \theta\cos\theta}$

Somebody cheated the Quotient Rule in §2.2.

#27 $f(t) = \dfrac{1-2t}{t+3} = -\dfrac{2t-1}{t+3}$

If asked to use the DEFINITION of the derivative to find $f'(t)$, then it's $\dfrac{f(t+h)-f(t)}{h}$ time.

$$\frac{f(t+h)-f(t)}{h} = -\left(\frac{1}{h}\left(\frac{2(t+h)-1}{t+h+3} - \left(\frac{2t-1}{t+3}\right)\right)\right)$$

$$= -\frac{1}{h}\left[\frac{(2t+2h-1)(t+3)-(2t-1)(t+h+3)}{(t+3)(t+h+3)}\right]$$

$$= -\frac{1}{h}\left[2t^2+6t+2ht+6h-t-3-2t^2-2ht-6t+t+h+3\right]$$

LCD

$$= -\frac{1}{h}\left[\frac{\cancel{2ht} + 6h \cancel{-2ht} + h}{LCD}\right] = -\frac{1}{h}\left[\frac{h[\quad +6 \quad +1]}{LCD}\right]$$

$$= -\cancel{\tfrac{1}{h}}\left[\frac{7}{(t+3)(t+3+h)}\right] \xrightarrow{h \to 0} \boxed{-\left[\frac{7}{(t+3)^2}\right] = f'(t)}$$

$h \neq 0$

Check w/ quotient rule $\quad \dfrac{f'g - fg'}{g^2}$

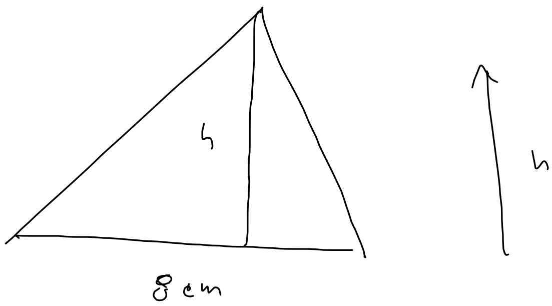
$$\frac{1-2t}{t+3} = f(t) \Rightarrow f'(t) = \frac{-2(t+3) - (1-2t)(1)}{(t+3)^2}$$

$$= \frac{-2t-6-1+2t}{(t+3)^2} = \frac{-7}{(t+3)^2}$$

Quotient Rule $\underline{\text{is}}$ Product Rule.

$$\left(\frac{f}{g}\right)' = \left(f g^{-1}\right)' = f' g^{-1} + f(-g^{-2})g'$$

$$= \frac{f'}{g} - \frac{fg'}{g^2} = \frac{f'g - fg'}{g^2}$$



$h = 4\text{cm} \pm .1\text{cm}$

$A = \frac{1}{2}bh = 4h$ $h=4,\ dh = .1$

$\frac{dA}{dh} = 4$ $dy = f'(x)\,dx$

$dA = 4dh = .4$ estimate of error.

Actual ERROR:

$A(4.1) - A(4.0) = 4(4.1) - 4(4) = 16.4 - 16 = .4$ is exact error

Relative Error: $\frac{\Delta A}{A} \approx \frac{dA}{A} = \frac{.4}{16} = \frac{.1}{4} = .025$

Percent Error: Previous times 100%.

$= 2.5\%$

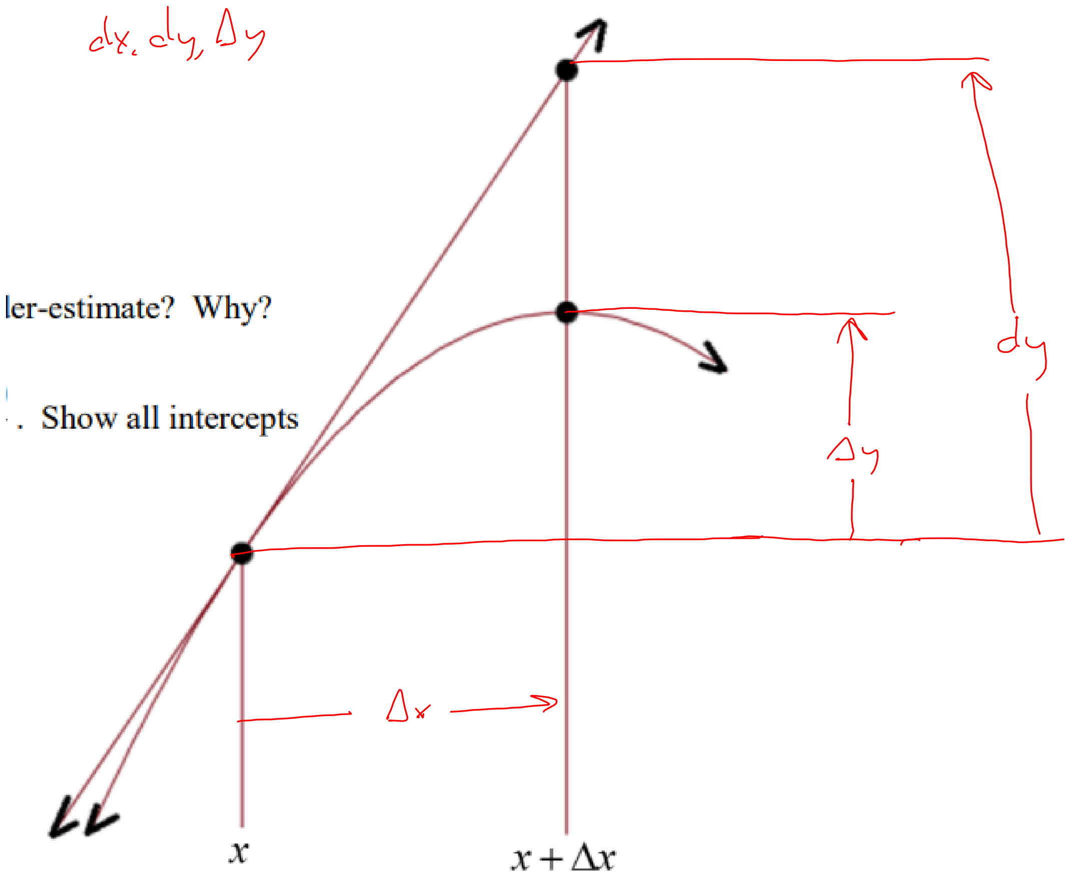


Figure for Bonus #2

§2.9 idea

$$\frac{dy}{dx} = f'(x)$$

Test scheduled

$$dy = f'(x)\,dx$$

$$y - y_1 = m(x - x_1)$$

$$\underbrace{}_{\Delta y} = m\,\Delta x$$

$$f(x) - f(x_1) \approx f'(x)\boxed{(x - x_1)}$$

Tan Line.

$$\Delta y \approx dy = f'(x)\,\Delta x$$