§2.1 #s 3, 10, 22

(3a) $f(x) = 4x - x^2$ $\qquad - (x^2 - a^2)$

(i) $\dfrac{f(x) - f(a)}{x-a} = \dfrac{4x - x^2 - [4a - a^2]}{x-a} = \dfrac{4x - 4a - x^2 + a^2}{x-a}$

$= \dfrac{4(x-a) - (x-a)(x+a)}{x-a} = \dfrac{(x-a)[4 - (x+a)]}{x-a}$

$= \boxed{\begin{matrix} 4 - x - a \\ (x \neq a) \end{matrix}} \xrightarrow{x \to a} 4 - a - a = 4 - 2a = f'(a) \Rightarrow f'(1) = 4 - 2(1) = 2 = m_{tan}\Big|_{x=1}$

$(x^2 + 2xh + h^2)$

(ii) $\dfrac{f(x+h) - f(x)}{h} = \dfrac{4(x+h) - (x+h)^2 - (4x - x^2)}{h}$

$= \dfrac{\cancel{4x} + 4h \cancel{- x^2} - 2xh - h^2 \cancel{- 4x} \cancel{+ x^2}}{h} = \dfrac{4h - 2xh - h^2}{h}$

$= \dfrac{h(4 - 2x - h)}{h} = \underset{(h \neq 0)}{4 - 2x - h} \xrightarrow{h \to 0} 4 - 2x = f'(x)$

$\Rightarrow f'(1) = 4 - 2(1) = 2 = m_{tan}\Big|_{x=1}$

(b) $y = m(x - x_1) + y_1$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Let (x, y) on the line.

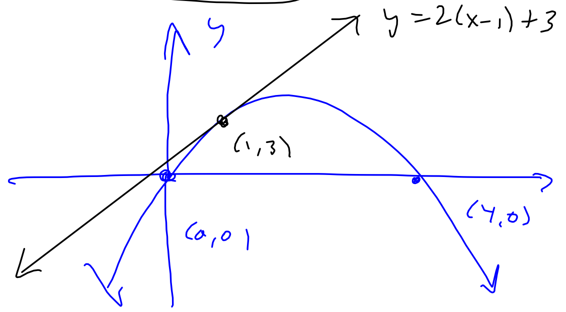
Then $m = \dfrac{y - y_1}{x - x_1} = m \Rightarrow y - y_1 = m(x - x_1)$

$\Rightarrow \underline{y = m(x - x_1) + y}$

(b) $y = f'(1)(x-1) + f(1)$

$\boxed{y = 2(x-1) + 3}$

(c)



(10) $(4, \frac{1}{2}) = (x_1, y_1) = (x_1, f(x_1))$

$(a-b)(a+b) = a^2 - b^2$

$f(x) = \frac{1}{\sqrt{x}}$

$$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h} = \frac{1}{h}\left[\frac{1}{\sqrt{x+h}}\cdot\frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}}\cdot\frac{\sqrt{x+h}}{\sqrt{x+h}}\right]$$

$$= \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right] = \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}\right]\left[\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right]$$

$$= \frac{1}{h}\left[\frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x+h}+\sqrt{x})}\right] = \frac{1}{\cancel{h}}\left[\frac{-\cancel{h}}{\sqrt{x}\sqrt{x+h}(\sqrt{x+h}+\sqrt{x})}\right]$$

$$= \frac{-1}{\sqrt{x}\sqrt{x+h}(\sqrt{x+h}+\sqrt{x})} \xrightarrow{h \to 0} \frac{-1}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})}$$

$(h \neq 0)$

$$= \frac{-1}{x(2\sqrt{x})} = \frac{-1}{2x\sqrt{x}} = \boxed{\frac{-1}{2x^{3/2}} = f'(x)}$$

$2x \cdot x^{\frac{1}{2}} = 2x^{\frac{3}{2}}$ $\Rightarrow f'(a) = \frac{-1}{2a^{3/2}}$

$$f'(4) = \frac{-1}{2(4)^{3/2}} = \frac{-1}{2(2)^3} = \frac{-1}{16}$$

$$4^{\frac{3}{2}} = (4^3)^{\frac{1}{2}} = (4^{\frac{1}{2}})^3 = 2^3 = 8$$

$$\boxed{(\sqrt{x})^2 = x \qquad \sqrt{x^2} = |x|}$$

(e) $(4, \frac{1}{2})$:

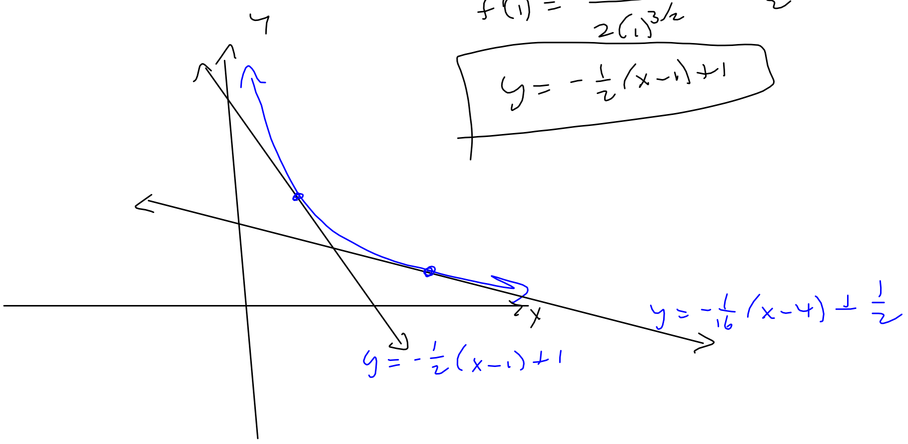
$y = m(x - x_1) + y_1$

$$\boxed{y = \frac{-1}{16}(x-4) + \frac{1}{2}}$$

(9) $(1,1) = (x_1, y_1)$

$$f'(1) = \frac{-1}{2(1)^{3/2}} = -\frac{1}{2}$$

$$\boxed{y = -\frac{1}{2}(x-1)+1}$$



22 $y=f(x)$ thru $(4,3)$ w/ $f'(4)=?$, $f(4)=?$

Tangent to $f(x)$ @ $(4,3)$ contains $(0,2)$.

I did this:

$y=f'(4)(x-4)+f(4)$

$y=f'(4)(x-4)+3$ from $(4,f(4))=(4,3)$

$\Rightarrow$ $2=f'(4)(0-4)+3$ from $(0,2)$ on $L_4(x)$.

$2=-4f'(4)+3$

$-1=-4f'(4)$

$\boxed{\frac{1}{4}=f'(4)}$

$(4,3)$ & $(0,2)$

on $L_4(x)$ $\Rightarrow$

$f'(4)=\frac{2-3}{0-4}=\frac{-1}{-4}=\boxed{\frac{1}{4}=f'(4)}$

$$\frac{d}{dx}\left[f(g(x))\right] = \frac{df}{dg}\cdot\frac{dg}{dx}$$

$f'(g(h(x))) =$

$= \frac{1}{3}\sin(x^2-7x)^{-2/3}$

$k(x) = \sqrt[3]{\sin(x^2-7x)}$

$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} \Longrightarrow f'(x) = \frac{1}{3}x^{-2/3} \Longrightarrow f'(g(x)) = \frac{1}{3}g(x)^{-2/3}$

$g(x) = \sin(x) \Longrightarrow g'(x) = \cos(x) \Longrightarrow g'(h(x)) = \cos(h(x))$

$= \cos(x^2-7x)$

$h(x) = x^2-7x \Longrightarrow h'(x) = 2x-7$

$\frac{df}{dg}\cdot\frac{dg}{dh}\cdot\frac{dh}{dx}$

$f'(g(x)) = \frac{1}{3}\sin(x)^{-2/3}$

$f'(g(h(x)))\, g'(h(x))\, h'(x)$

$= \frac{1}{3}\left(\sin(x^2-7x)\right)^{-\frac{2}{3}}\left(\cos(x^2-7x)\right)(2x-7)$

$k(x) = \sqrt[3]{\sin(x^2-7x)} = \left(\sin(x^2-7x)\right)^{\frac{1}{3}} \Longrightarrow$

$k'(x) = \frac{1}{3}\left(\sin(x^2-7x)\right)^{-\frac{2}{3}}\left(\cos(x^2-7x)\right)(2x-7)$

$\frac{d}{dx}\left[fg\right] =$

$(fg)' = f'g + fg'$

$5x^2y^2 - 7xy^3 = x^2\sin(xy)$

Find $\frac{dy}{dx}$, by assuming y is a function of x

$\frac{d}{dx}[x] = 1$

$\frac{d}{dx}[y] = y'$

$5(2x)y^2 + 5x^2(2yy') - 7(1)y^3 - 7x(3y^2y')$

$= 2x\sin(xy) + x^2\cos(xy)(1y + xy')$

$10x^2yy' - 21xy^2y' - x^3\cos(xy)y'$ $\quad$ $x^2y\cos(xy) + x^3\cos(xy)y'$

$= 10xy^2 - 7y^3 + 2x\sin(xy) + x^2y\cos(xy)$

$$y' = \frac{\text{STUFF}}{10x^2y - 21xy^2 - x^3\cos(xy)}$$

$-x^3\cos(xy)$