§2.5 #37

$$\frac{d}{d\theta}\left[(\cot(\sin\theta))^2\right] = 2(\cot(\sin\theta))'(-\csc^2(\sin\theta))(\cos\theta)$$

$$(\cot\theta)' = \left(\frac{\cos\theta}{\sin\theta}\right)' = \frac{-\sin\theta\sin\theta - (\cos\theta)(\cos\theta)}{\sin^2\theta} = \frac{-1}{\sin^2\theta} = -\csc^2\theta$$

$$\frac{d}{d\cot(\sin\theta)}\left[(\cot(\sin\theta))^2\right]\frac{d}{d\sin\theta}\left[\cot(\sin\theta)\right]\frac{d}{d\theta}\left[\sin\theta\right]$$

$$\left[2\cot(\sin\theta)\right]\cdot\left[-\csc^2(\sin\theta)\right]\cdot\left[\cos\theta\right]$$

$(\cot(\sin\theta))^2 = f(g(h(\theta)))$, where

$f(x) = x^2$, $g(x) = \cot(x)$, $h(x) = \sin(x)$

$$\frac{d}{dx}\left[f(g(h(x)))\right] = \frac{df}{dg}\cdot\frac{dg}{dh}\cdot\frac{dh}{dx}$$

$$\left[2g^2\right]\left[-\csc^2(h)\right]\left[\cos(x)\right]$$

$$= (2\cot(\sin(x)))(-\csc^2(\sin(x)))(\cos x)$$

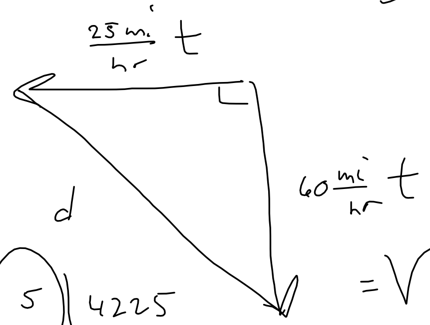
$$\frac{d}{dx}\left[(2x+5)^2\right] = 2(2x+5)'(2)$$

S'2.8 #17

Car 1: Travels south @ 60 $\frac{mi}{hr}$

Car 2: 〃 west @ 25 $\frac{mi}{hr}$

Find rate the $\underbrace{\text{distance}}_{D}$ between them is increasing



Want $\frac{dD}{dt}$

$$D = \sqrt{(25t)^2 + (60t)^2}$$

$$= \sqrt{625t^2 + 3600t^2} = \sqrt{4225t^2}$$

$$= 5 \cdot 13 \, |t|$$

$$= 65t \text{, because } t \geq 0.$$

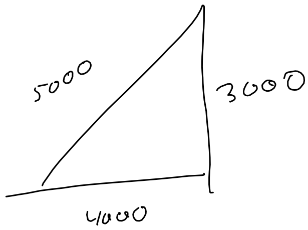
5 | 4225
5 | 845
13 | 169
13

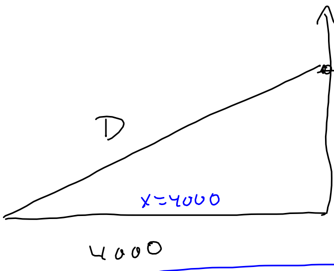
Now $\frac{dD}{dt} = \boxed{65 \ \frac{mi}{hr}}$

43. A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft.

(a) How fast is the distance from the television camera to the rocket changing at that moment?

(b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?





Lexicon –

t = time in seconds

a

$$D = \sqrt{4000^2 + \left(\left(600 \tfrac{ft}{sec}\right)(t)\right)^2} = \sqrt{4000^2 + 600^2 t^2} = \left(4000^2 + 600^2 t^2\right)^{\frac{1}{2}}$$

t sec

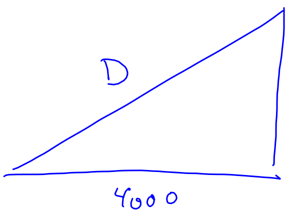
$$\left.\frac{dD}{dt}\right|_{y=3000} = \boxed{\tfrac{1}{2}\left(4000^2 + 600^2 t^2\right)^{-\frac{1}{2}}\left(2(600)^2 t\right)} = \boxed{\frac{600^2 t}{\sqrt{4000^2 + 600^2 t^2}}}$$

very specific

use all our info.

But I don't know when $y = 3000$ ft.

$$D = \sqrt{4000^2 + 600^2 t^2}$$



$\frac{dy}{dt} = 600$ when $y = 3000$ ft

$\frac{dx}{dt} = 0$

$$\cancel{\left.\frac{dD}{dt}\right|_{y=3000} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(600 \tfrac{ft}{sec}\right)^2} = \sqrt{0^2 + 600^2} = \boxed{600 \tfrac{ft}{sec}}}$$

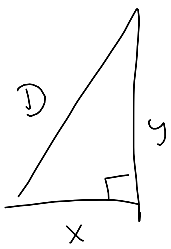
$\left.\frac{dy}{dt}\right|_{y=3000}$

$$D = \left(4000^2 + 600^2 t^2\right)^{\frac{1}{2}}$$

Suggestions?

I'll make a video after lunch.

$$\left.\frac{dD}{dt}\right|_{y=3000} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{0^2 + 600^2} = 600$$

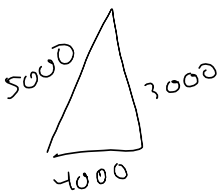


$$\left.\frac{dD}{dt}\right|_{y=300} = \frac{1}{2}(3000 +$$

$$D = \left(x^2 + y^2\right)^{\frac{1}{2}} = \left(4000^2 + y^2\right)^{\frac{1}{2}}$$

$$\left.\frac{dD}{dt}\left[\quad\right]\right|_{y=3000} = \frac{1}{2}\left(4000^2 + y^2\right)^{-\frac{1}{2}}\left(2y\frac{dy}{dt}\right)$$

$$= \frac{1}{\sqrt{(4000)^2 + (3000)^2}}(3000)(600)$$



$$= \frac{1}{\sqrt{5000^2}}(3000)(600)$$

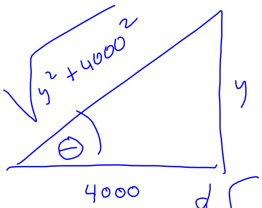
$$= \left(\frac{1}{5000}\right)(1800000) = \frac{1800}{5} = 360$$

43. A television camera is positioned 4000 ft from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Also, the mechanism for focusing the camera has to take into account the increasing distance from the camera to the rising rocket. Let's assume the rocket rises vertically and its speed is 600 ft/s when it has risen 3000 ft.

(a) How fast is the distance from the television camera to the rocket changing at that moment?

(b) If the television camera is always kept aimed at the rocket, how fast is the camera's angle of elevation changing at that same moment?

Want $\left.\frac{d\theta}{dt}\right|_{y=3000}$



$\frac{d}{dt}\left[\tan\theta = \frac{y}{4000}\right]$

$\sec^2\theta$

$f(x) = \tan(x)$ $\quad f(t) = \tan(t)$

$g(x) = \theta(x)$ $\quad \theta(t) = \theta(t)$

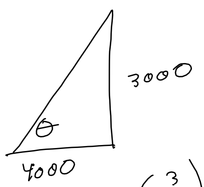
So $\frac{d}{dt}[f(g(t))] = \frac{df}{dg}\cdot\frac{dg}{dt} = (\sec^2\theta)\left(\frac{d\theta}{dt}\right)$

$\frac{d}{dt}\left[\tan\theta = \frac{y}{4000}\right] \Rightarrow (\sec^2\theta)\,\theta' = \frac{1}{4000}y'$

$\frac{dy}{dt} = y'$

$\Rightarrow \theta' = \frac{\frac{1}{4000}y'}{\sec^2\theta}$

$= \frac{\left(\frac{1}{4000}\right)(600)}{\sec^2\theta}$

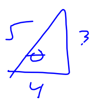


$\theta = \arctan\left(\frac{3}{4}\right)$

$= \frac{\frac{600}{4000}}{\sec^2(\arctan(3/4))}$

$\frac{3}{20\left(\frac{5}{4}\right)^2} = \frac{3}{\cancel{20}_5}\cdot\frac{\cancel{16}^4}{25} = \frac{12}{125}\ \frac{\text{radians}}{\text{sec}}$ $\quad \frac{600}{4000} = \frac{6}{40} = \frac{3}{20}$

$\approx .096\ \frac{\text{rad}}{\text{s}}$?



$\frac{\frac{2}{3}}{\frac{1}{2}} = \frac{2}{3}\cdot\frac{2}{1}$

$\frac{\frac{A}{B}}{\arctan\theta} = \frac{\frac{A}{B}}{\frac{\arctan\theta}{1}} = \left(\frac{A}{B}\right)\left(\frac{1}{\arctan\theta}\right)$

$\frac{\frac{2}{3}}{2} = \frac{\frac{2}{3}}{\frac{2}{1}} = \frac{2}{3}\cdot\frac{1}{2} = \frac{1}{3}$

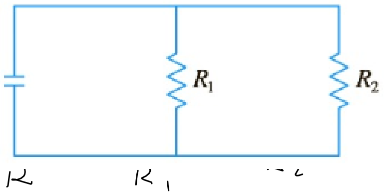
39. If two resistors with resistances R_1 and R_2 are connected in parallel, as in the figure, then the total resistance R, measured in ohms (Ω), is given by

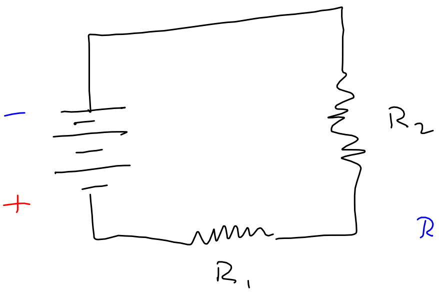
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R_1 and R_2 are increasing at rates of 0.3 Ω/s and 0.2 Ω/s, respectively, how fast is R changing when $R_1 = 80\ \Omega$ and $R_2 = 100\ \Omega$?

Want!

$$\left.\frac{dR}{dt}\right|_{\substack{R_1 = 80\Omega \\ R_2 = 100\Omega}}$$





$$\frac{d}{dt}\left[R^{-1} = R_1^{-1} + R_2^{-1}\right]$$

$$-1R^{-2}\frac{dR}{dt} = -1R_1^{-2}\frac{dR_1}{dt} - 1R_2^{-2}\frac{dR_2}{dt}$$

$$-R^{-2}R' = -R_1^{-2}R_1' - R_2^{-2}R_2'$$

$$R' = \left(\frac{R_1'}{R_1^2} - \frac{R_2'}{R_2^2}\right)(-R^2)$$

$$\left.R'\right|_{\substack{R_1 = 80\Omega \\ R_2 = 100\Omega}} =$$

Oops - forgot the square.

$$\left(\frac{.3}{80^2} - \frac{.2}{100^2}\right)(-R^2)$$

$$\cancel{= \left(\frac{.3}{80} + \frac{.2}{100}\right)R^2 = \left(\frac{.3(100) + .2(80)}{(80)(100)}\right)\left(\frac{400}{9}\right)}$$

$$\cancel{\frac{1}{R} = \frac{1}{80} + \frac{1}{100} = \frac{100+80}{(100)(80)} = \frac{180}{8000} = \frac{9}{400} \Rightarrow R = \frac{400}{9}}$$

$$\cancel{\frac{46 \cdot 4}{80 \cdot 9} = \frac{23}{90}}$$

$$\left(\frac{.3}{80^2} + \frac{.2}{100^2}\right)\left(\frac{400}{9}\right)^2 = \left(\frac{.3(100)^2 + .2(80)^2}{(80^2)(100)^2}\right)\left(\frac{400^2}{9^2}\right)$$

See Maple Notes for this date.

$$\boxed{\approx 0.1320987654\ \frac{\Omega}{\sec}}$$