21. The force F acting on a body with mass m and velocity v is the rate of change of momentum: $F = (d/dt)(mv)$. If m is constant, this becomes $F = ma$, where $a = dv/dt$ is the acceleration. But in the theory of relativity the mass of a particle varies with v as follows: $m = m_0/\sqrt{1 - v^2/c^2}$, where m_0 is the mass of the particle at rest and c is the speed of light. Show that

$$F = \frac{m_0 a}{(1 - v^2/c^2)^{3/2}} \qquad \frac{dv}{dt}$$

Do we ignore the v^2 when we differentiate? I did!

$(1 - \frac{v^2}{c^2})^{3/2}$ $\quad F = \frac{m_0 a}{(1 - v^2/c^2)^{3/2}}$

$$F = \frac{d}{dt}\left[mv\right] = \frac{d}{dt}\left[\frac{m_0}{(1-v^2/c^2)^{1/2}}v\right] = \frac{d}{dt}\left[m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}v\right]$$

$$= \underline{m_0\left(-\tfrac{1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}}\underline{(v)} + \underline{m_0\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}}\underline{\frac{dv}{dt}}$$

$$\doteq \frac{-m_0 v}{2\left(1-\frac{v^2}{c^2}\right)^{3/2}} + \frac{m_0 a}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \cdot \frac{\left(1-\frac{v^2}{c^2}\right)\cdot 2}{\left(1-\frac{v^2}{c^2}\right)\cdot 2}$$

$$= \frac{-m_0 v + 2m_0 a\left(1-\frac{v^2}{c^2}\right)}{2\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

isn't getting us home.

$$\frac{d}{dt}\left[\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}\right] = \frac{1}{2}\left(1-\frac{v^2}{c^2}\right)\boxed{\left(\frac{-2vv'}{c^2}\right)}$$

Go Beast Mode $\quad (fg)' = f'g + fg'$

$$\frac{d}{dt}\left[\frac{m_o v}{(1-v^2/c^2)^{1/2}}\right] = \frac{d}{dt}\left[m_o \left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}(v)\right]$$

$$= m_o \cancel{\left(-\frac{1}{2}\right)}\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}\left(-\frac{\cancel{2}\, v v'}{c^2}\right)(v) + m_o\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}(v')$$

$$= m_o\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} v'\left[\left(1-\frac{v^2}{c^2}\right)^{-1}\frac{v^2}{c^2} + 1\right]$$

$$= m_o\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} a\left[\frac{v^2}{c^2\left(1-\frac{v^2}{c^2}\right)} + \frac{c^2\left(1-\frac{v^2}{c^2}\right)}{c^2\left(1-\frac{v^2}{c^2}\right)}\right]$$

$$= m_o\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} a\left[\frac{v^2+c^2-v^2}{c^2\left(1-\frac{v^2}{c^2}\right)}\right] = \frac{m_o a}{\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}}}\left[\frac{c^2}{c^2\left(1-\frac{v^2}{c^2}\right)}\right]$$

$$= \frac{m_o a}{(1-v^2/c^2)^{1+\frac{1}{2}}} = \frac{m_o a}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$ is what we want.

$$-\frac{3}{2}+\frac{1}{2} = \frac{2}{2} = -1$$

$$\frac{\left(1-\frac{v^2}{c^2}\right)^{-\frac{3}{2}}}{\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}}} = \left(1-\frac{v^2}{c^2}\right)^{-1}$$

How much metal to cover a 5-meter sphere with 5mm-thick sheet metal cover?

$V = \frac{4}{3}\pi r^3$ = volume

$\frac{4}{3}\pi\left[5.005^3 - 5^3\right] = \Delta V$ = Actual amount

$\boxed{\Delta V \approx 1.5723675} \approx dV = \frac{dV}{dr} \cdot dr = \frac{dV}{dr}\Delta r$

$dr \equiv \Delta r$ = increment of the independent variable

One way to view it

$\frac{\Delta V}{\Delta r} \approx \frac{dV}{dr} \Rightarrow \Delta V \approx \frac{dV}{dr}\Delta r$

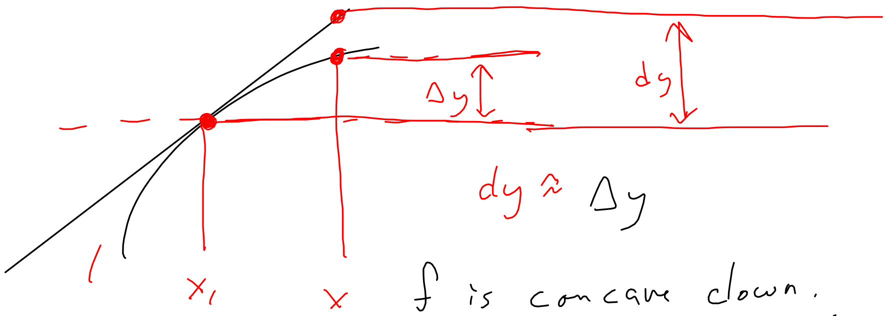
Another way to view it

$V \approx \left.\frac{dV}{dr}\right|_{r=5}(r-5) + V(5)$

$f(x) \approx y = m(x - x_1) + y_1$

$$f(x) \approx L(x) = f'(x_1)\underbrace{(x - x_1)}_{\Delta x} + f(x_1)$$

$f(x) - f(x_1) = \Delta y \approx f'(x_1)\Delta x \equiv dy$ is the definition



$dy \approx \Delta y$

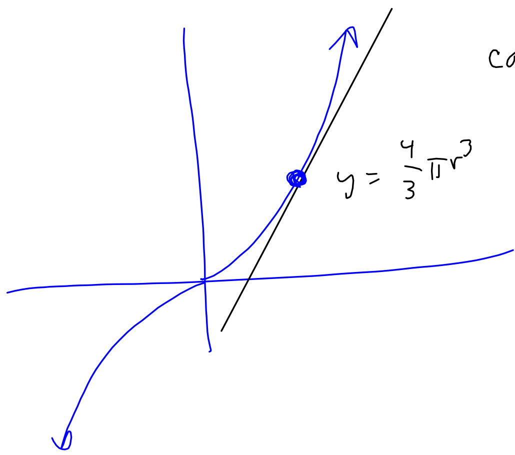
f is concave down.
so $L(x)$ lives above it

and $dy > \Delta y$

$$V = \frac{4}{3}\pi r^3 \Rightarrow dV = 4\pi r^2 \underbrace{dr}_{=\Delta r} = 4\pi (5)^2 (.005)$$

$\boxed{\approx 1.57\ m^3}$

$\approx$ 1.570796327 slightly Less than ΔV.



concave up & $L(x)$
is BELOW the curve.
& $\Delta V > dV$