56. (a) The curve $y = |x|/\sqrt{2 - x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point (1, 1).

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

S25

$\boxed{\begin{array}{c}x\text{-int} \\ x=0\end{array}}$

$$f(x) = \frac{|x|}{\sqrt{2-x^2}} = \begin{cases} \dfrac{x}{\sqrt{2-x^2}} & \text{if } x \geq 0 \\ \dfrac{-x}{\sqrt{2-x^2}} & \text{if } x < 0 \end{cases}$$

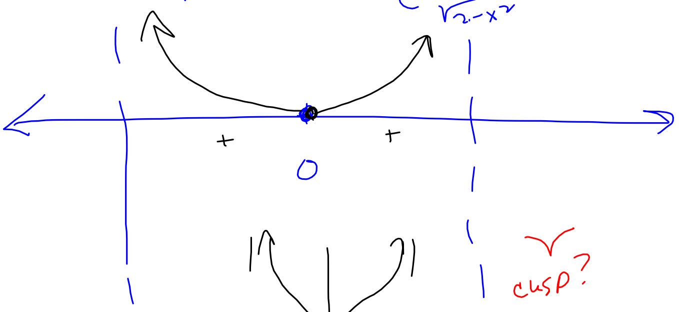
$2 - x^2 = 0$

$x^2 = 2$

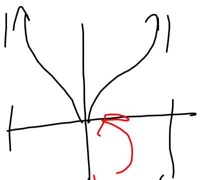
$x = \pm\sqrt{2}$

vert. Asymp.

V.A.



cusp?



$y = \sqrt{2}$

Pointy

Domain:

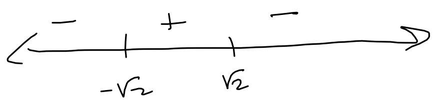
Need $2 - x^2 \geq 0$ & $2 - x^2 \neq 0$

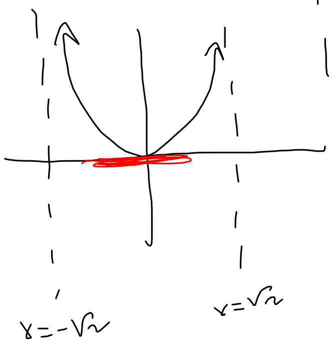
$\frac{\text{stuff}}{0}$

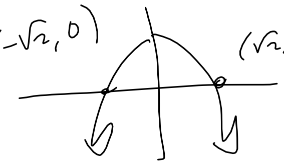
$2 - x^2 > 0$

$(\sqrt{2} - x)(\sqrt{2} + x)$

$= -(x - \sqrt{2})(x + \sqrt{2})$







$$f(x) = \frac{|x|}{\sqrt{2-x^2}} = \begin{cases} \dfrac{x}{\sqrt{2-x^2}} & \text{if } x \geq 0 \\ \dfrac{-x}{\sqrt{2-x^2}} & \text{if } x < 0 \end{cases}$$

$(fg)' = f'g + fg'$

$x \geq 0$: $f(x) = x\,(2-x^2)^{-\frac{1}{2}}$

$f = x$

$g = (2-x^2)^{-\frac{1}{2}}$

$\Rightarrow f'(x) = 1\,(2-x^2)^{-\frac{1}{2}} + x \cdot x\,(2-x^2)^{-\frac{3}{2}}$

$f' = 1$

$g' = -\frac{1}{2}(2-x^2)^{-\frac{3}{2}}(-2x)$

$= \dfrac{1}{\sqrt{2-x^2}} + \dfrac{x^2}{\sqrt{2-x^2}^{\,3}}$

$= \dfrac{x}{(2-x^2)^{3/2}}$

$= \dfrac{1}{\sqrt{2-x^2}} \cdot \dfrac{\sqrt{2-x^2}^{\,2}}{\sqrt{2-x^2}^{\,2}} + \dfrac{x^2}{\sqrt{2-x^2}^{\,3}}$

$= \dfrac{x}{\left((2-x^2)^3\right)^{\frac{1}{2}}}$

$= \dfrac{2-x^2+x^2}{(2-x^2)^{3/2}} = \dfrac{2}{(2-x^2)^{3/2}}$

$= \dfrac{x}{\left(\sqrt{2-x^2}\right)^3}$

From the right.

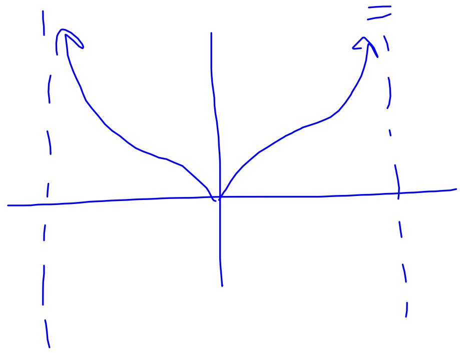
Note: from the left, f(x) changes to

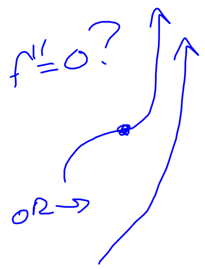
$$-\frac{x}{\sqrt{2-x^2}} \xrightarrow{\frac{d}{dx}} -\frac{2}{(2-x^2)^{3/2}}$$

$$f'_-(0) = -\frac{2}{2^{3/2}} = -\frac{1}{\sqrt{2}}$$

$$f'_+(0) = +\frac{2}{2^{3/2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

It is a cusp sort of deal





56. (a) The curve $y = |x|/\sqrt{2-x^2}$ is called a *bullet-nose curve*. Find an equation of the tangent line to this curve at the point (1, 1).

(b) Illustrate part (a) by graphing the curve and the tangent line on the same screen.

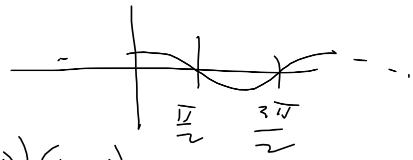
(a) $f(1) = \frac{1}{\sqrt{2-x^2}} = \frac{1}{\sqrt{1}} = 1$ $\boxed{(x_1, y_1) = (1, 1)}$

$f'(1) = \frac{2}{\sqrt{2-1^2}} = \frac{2}{1} = 2 = m_{TAN}$

$y = m_{TAN}(x - x_1) + y_1$ $\boxed{y = 2(x-1)+1}$

§2.4 #34

$\cos x = 0$



$$f(x) = \frac{\cos x}{\sin x + 2}$$

$$f'(x) = \frac{-\sin(x)\,(\sin(x)+2) - (\cos(x))(\cos x)}{(\sin(x)+2)^2}$$

$\sin(x) + 2 = 0$

$$= \frac{-\sin^2 x - 2\sin x - \cos^2 x}{(\quad)^2}$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$= \frac{-(\sin^2 x + \cos^2 x) - 2\sin x}{(\quad)^2}$$

$$\frac{d}{dx}\left[\frac{x}{(2-x^2)^{1/2}}\right]$$

$$= \frac{1\,(2-x^2)^{\frac{1}{2}} - x\left(\frac{1}{2}\right)(2-x^2)^{-\frac{1}{2}}(-2x)}{\left((2-x^2)^{1/2}\right)^2}$$

$(2-x^2)$

$$= \frac{-1-2\sin x}{(\quad)^2}$$

$$= \frac{-(2\sin x + 1)}{(\sin(x)+2)^2} \overset{\text{SET}}{=} 0$$

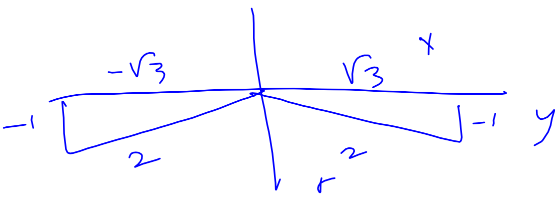
$=$ same!

$\Longrightarrow$ $2\sin x + 1 = 0$

$2\sin x = -1$

$\sin x = -\frac{1}{2}$ $\Longrightarrow$ $x \in \left\{\frac{7\pi}{6}, \frac{11\pi}{6}\right\}$ if $x \in [0, 2\pi)$

Find ALL sol'ns!



$$\boxed{\left\{ x \;\middle|\; x = \frac{7\pi}{6} + 2n\pi \text{ OR } x = \frac{11\pi}{6} + 2n\pi,\; n \in \mathbb{Z} \right\}}$$

$\sqrt{1-\frac{v^2}{c^2}}$ Lorentz

The problem I wanted to look at was #34 in Section 2.7. I went right past it.
That's a good one to look at.