Tangent line to $\sin(x)$ @ $x = \frac{\pi}{3}$

$(x_1, y_1) = \left(\frac{\pi}{3}, \sin\left(\frac{\pi}{3}\right)\right) = \left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

$f'(x) = \cos(x)$

$f'\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} = m_{tan}$

$y = f'(x_1)(x - x_1) + f(x_1)$

$L_{\frac{\pi}{3}}(x) = f(x_1) + f'(x_1)(x - x_1)$ Book

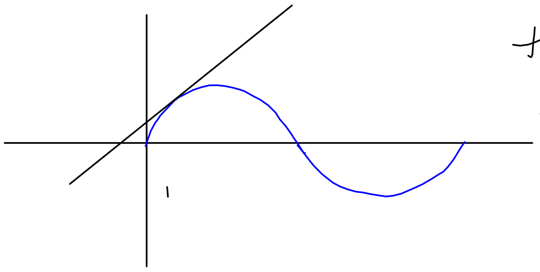
$= \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

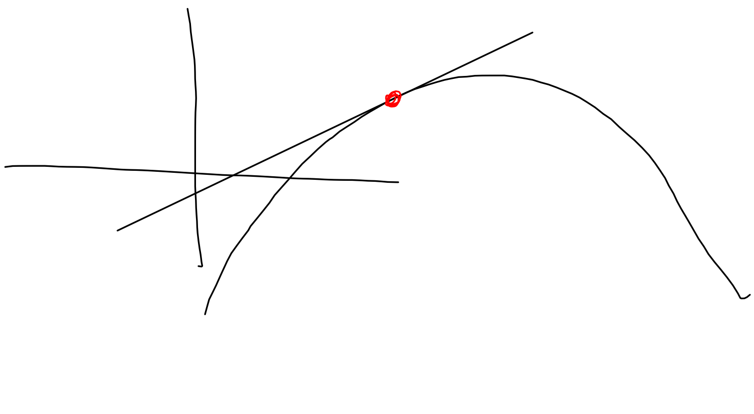
Check

$L(0) = \frac{1}{2}\left(-\frac{\pi}{3}\right) + \frac{\sqrt{3}}{2}$

$\approx$

Should've known the y-int was positive from the picture, already.





Quick Application

Falling body

$$s = -\frac{1}{2} g t^2 + v_0 t + s_0$$

g = acceleration of gravity $= 9.8 \frac{m}{s^2}$

v_0 = initial velocity

s_0 = " height

Thrown upward at $50\ m/s = v_0$

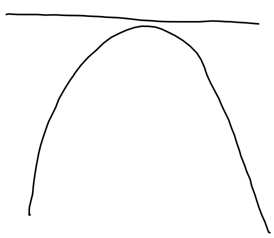
from a height of 10m.

What's the maximum height?

When does the ball hit the ground? $\boxed{\text{Set } s(t) = 0}$

$$s(t) = -\frac{1}{2}(9.8)t^2 + 50t + 10$$

$$= -4.9t^2 + 50t + 10$$



$$s'(t) = -9.8t + 50 \overset{SET}{=} 0$$

$$-9.8t = -50$$

$$t = \frac{-50}{-9.8} = \frac{+50}{\frac{98}{10}} = \frac{500}{98} = \frac{250}{49} \approx 5.102040816$$

& $s(5.102040816) \approx 137.5510204$ meters.

College Algebra

$$-4.9t^2 + 50t + 10$$

$$-\frac{b}{2a} = \text{x-coord of vertex}$$

$$= \frac{-50}{2(-4.9)} = \frac{50}{9.8}$$

$$\begin{array}{r} {}^{3}4.9 \\ 40 \\ \hline 196.0 \end{array}$$

$a = -4.9,\ b = 50,\ c = 10$

$$b^2 - 4ac = (50)^2 - 4(-4.9)(10)$$

$$= 2500 + 196 = 2696$$

$$t = \frac{-50 \pm \sqrt{2696}}{2(-4.9)} \quad \begin{array}{l} \nearrow 10.40030816 \\ \searrow -0.1962265248 \end{array}$$

Find the midpoint:

$$\frac{10.4\ldots + -.196\ldots}{2} \approx 5.102040816$$

$$\lim_{t\to 0} \frac{\sin(5t)}{\sin(3t)} = \underline{\lim_{t\to 0}\left[\frac{\sin(5t)}{5t}\cdot\frac{3t}{\sin(3t)}\cdot\frac{5}{3}\right]}$$

$$= 1\cdot 1\cdot \frac{5}{3}$$

is plenty of work

$$t\to 0 \Rightarrow 5t = u \to 0$$

$$t\to 0 \Rightarrow 3t = v \to 0$$

$$\underbrace{\frac{5}{3}\lim_{u\to 0}\left[\frac{\sin(u)}{u}\right]\lim_{v\to 0}\left[\frac{v}{\sin(v)}\right]}_{\text{underlying details.}} = \frac{5}{3}\cdot 1\cdot 1 = \frac{5}{3}$$

You can <u>NOT</u> factor out the '5' or the '3'!

$$\sin(5t) \neq 5\sin(t)$$

Chain Rule Practice

$$\frac{d}{dx}\left[\sin\left(\sqrt{\cos^2 x - 1}\right)\right]$$

$$= \cos\left(\sqrt{\cos^2 x - 1}\right)\left(\tfrac{1}{2}(\cos^2 x - 1)^{-\frac{1}{2}}\right)(2\cos(x))(-\sin(x))$$

$$(\cos^2 x - 1)^{\frac{1}{2}}$$

$$\underbrace{\cos^2 x - 1}_{g} \rightsquigarrow 2\cos(x)(-\sin(x))$$

$$(\cos x)^2 \rightsquigarrow 2\cos(x)(-\sin(x))$$

$$\frac{d}{dx}\left[\sqrt{\cos^2 x - 1}\right]$$

$$= \frac{d}{dx}\left[(\cos^2 x - 1)^{\frac{1}{2}}\right] = \frac{1}{2}\left[\cos^2 x - 1\right]^{-\frac{1}{2}}\left[2\cos(x)\right]\left[-\sin(x)\right]$$

Differentiate Implicitly to find $\frac{dy}{dx}$

$$\frac{d}{dx}\left[x^3 y^2 + 3xy^3 - \cos(xy) = 7x^2y^3\right]$$

$y^2 \rightsquigarrow 2yy'$ $\quad +\sin(xy)y \quad +\sin(xy)xy'$

$$\Longrightarrow 3x^2y^2 + 2x^3yy' + 3y^3 + 9xy^2y' - (-\sin(xy))(y + xy')$$

$$= 14xy^3 + 21x^2y^2y'$$

$$\left(2x^3y + 9xy^2 + x\sin(xy) - 21x^2y^2\right)y' = 14xy^3 - 3x^2y^2 - 3y^3 - y\sin(xy)$$

$$y' = \frac{14xy^3 - 3x^2y^2 - 3y^3 - y\sin(xy)}{2x^3y + 9xy^2 + x\sin(xy) - 21x^2y^2}$$

$$(7fg)' = 7[f'g + fg']$$

$$f = x^2, \quad g = y^3$$

$$(xy)' = x'y + xy' = 1y + xy'$$

$$\frac{dx}{dx} = 1$$