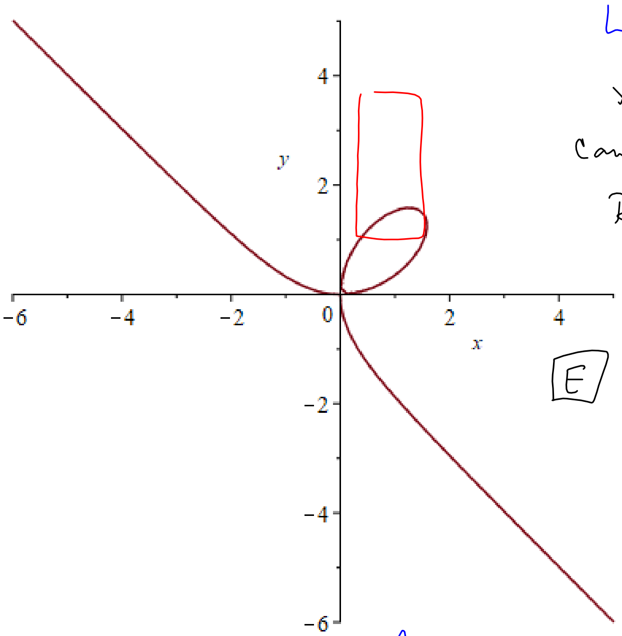


Locally a function

$x^3+y^3=3xy$

Can't solve for y

Recall Chain Rule

$$\frac{d}{dx}\left[f(g(x))\right]=\frac{df}{dg}\cdot\frac{dg}{dx}$$

$\boxed{E}$ $f(x)=\sin(x)$

$g(x)=x^2$

$f(g(x))=f(x^2)=\sin(x^2)$

$\cos(x^2)\cdot 2x$

$\frac{df}{dg}\cdot\frac{dg}{dx}$

General Power Rule

$$\frac{d}{dx}\left[g(x)^3\right]$$

$f(x)=x^3 \longrightarrow f'(x)=3x^2$

$g(x)=g(x)$

h

$$=\frac{df}{dg}\cdot\frac{dg}{dx}$$

$$=3g(x)^2\cdot g'(x)$$

Assume $y=f(x)$

Then $\frac{d}{dx}\left[y^3\right]=3y^2\cdot\frac{dy}{dx}=3y^2y'$

Last thing we need is product rule

$(fg)'=f'g+fg'$

$$\frac{d}{dx}\left[\left(x^2y^3\right)\right]=\frac{d}{dx}\left[x^2\right]y^3+x^2\frac{d}{dx}\left[y^3\right]$$

$$=2xy^3+x^2\left[3y^2y'\right]$$

$\boxed{\text{Assume } y=f(x)}$ at least locally.

$x^3+y^3=3xy$ Find $\frac{dy}{dx}$

$3x^2+3y^2y'=3y+3xy'$

$\frac{d}{dx}\left[x^3\right]=3x^2x'$
$=3x^2$

$3y^2y'-3xy'=3y-3x^2$

$x^2+y^2=1$

Find the slope at $x=\frac{1}{2}$

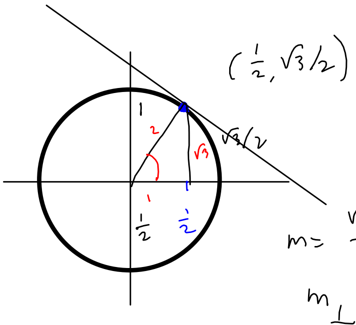
$x^2+y^2=1$ HARD WAY

$y^2 = 1-x^2$

$|y| = \sqrt{y^2} = \sqrt{1-x^2}$

$y = \pm\sqrt{1-x^2}$

$y = +\sqrt{1-x^2}$ is top $\frac{1}{2}$



$x=\frac{1}{2} \Rightarrow y = \sqrt{1-\left(\frac{1}{2}\right)^2} = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}$

$m = \frac{\sqrt{3}}{1}$

$m_\perp = -\frac{1}{m} = -\frac{1}{\sqrt{3}}$

$$f'(x) = \frac{dy}{dx} = \frac{d}{dx}\left[\sqrt{1-x^2}\right] = \frac{d}{dx}\left[(1-x^2)^{\frac{1}{2}}\right]$$

$$= \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x)$$

$$\Rightarrow f'(\tfrac{1}{2}) = \frac{1}{2}\left(1-(\tfrac{1}{2})^2\right)^{-\frac{1}{2}}\left(-2(\tfrac{1}{2})\right)$$

$$= \frac{1}{2}\left[\frac{3}{4}\right]^{-\frac{1}{2}}(-1)$$

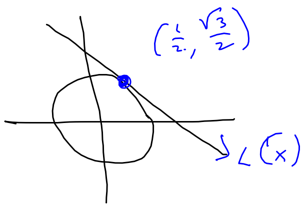
$$= -\frac{1}{2}\left[\frac{4}{3}\right]^{\frac{1}{2}} = -\frac{1}{2}\left[\frac{2}{\sqrt{3}}\right] = \boxed{-\frac{1}{\sqrt{3}} = m_\perp}$$

Tan line:

$x_1 = \frac{1}{2}, \; y_1 = \frac{\sqrt{3}}{2}, \; m = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$y = m(x-x_1)+y_1$

$$\boxed{L_{\frac{1}{2}}(x) = -\frac{1}{\sqrt{3}}\left(x-\frac{1}{2}\right)+\frac{\sqrt{3}}{2}}$$



Implicit Diff technique:

$$\frac{d}{dx}\left[x^2+y^2=1\right]$$

$2x+2yy'=0$

$2yy' = -2x$

$y' = \frac{-2x}{2y} = -\frac{x}{y}$

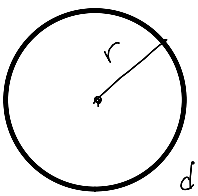
$x=\frac{1}{2}, \; y=\frac{\sqrt{3}}{2} \Rightarrow$

$$y' = -\frac{x}{y} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2}\cdot\frac{2}{\sqrt{3}} = -\frac{1}{\sqrt{3}} = m$$

$$y = -\frac{1}{\sqrt{3}}\left(x-\frac{1}{2}\right)+\frac{\sqrt{3}}{2}$$

An oil slick's radius is increasing at a rate of $\frac{10\text{ mi}}{\text{hr}}$. How fast is the area of the slick increasing when $r = 1$ mile?

Related Rates



want $\frac{dA}{dt}$

$$\frac{d}{dt}\left[r^2\right] = 2rr'$$

$$\frac{d}{dt}\left[A = \pi r^2\right]$$

$$A' = \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi r r'$$

$$A'(1) = 2\pi(1)(10) = \frac{20\pi \text{ mi}^2}{\text{hr}}$$