

$$f(x) = \frac{3x^2 - 13x - 10}{|x-5|} = \begin{cases} \frac{3x^2 - 13x - 10}{x-5} & \text{if } x \geq 5 \\ -\left(\frac{3x^2 - 13x - 10}{x-5}\right) & \text{if } x < 5 \end{cases}$$

$$|x-5| = \begin{cases} x-5 & \text{if } x-5 \geq 0 \\ -(x-5) & \text{if } x-5 < 0 \end{cases} = \begin{cases} x-5 & \text{if } x \geq 5 \\ -(x-5) & \text{if } x < 5 \end{cases}$$

$$3x^2 - 13x - 10 = (3x+2)(x-5)$$

$$a=3, b=-13, c=-10$$

$$\Rightarrow b^2 - 4ac = (-13)^2 - 4(3)(-10)$$

$$= 169 + 120$$

$$= 289 = 17^2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+13 \pm 17}{2(3)} = \frac{13 \pm 17}{6} \begin{cases} \frac{30}{6} = 5 \\ \frac{-4}{6} = -\frac{2}{3} \end{cases}$$

$$\Rightarrow 3x^2 - 13x - 10 = 3(x-5)\left(x + \frac{2}{3}\right) = (x-5)(3x+2)$$

$$\Rightarrow f(x) = \begin{cases} \frac{(3x+2)(x-5)}{x-5} & \text{if } x \geq 5 \\ -\frac{(3x+2)(x-5)}{x-5} & \text{if } x < 5 \end{cases} = \begin{cases} 3x+2 & \text{if } x \geq 5 \\ -(3x+2) & \text{if } x < 5 \end{cases}$$

$(x \neq 5)$

$$\Rightarrow f(x) \xrightarrow{x \rightarrow 5^+} 3(5)+2 = 17$$

$$f(x) \xrightarrow{x \rightarrow 5^-} -(3(5)+2) = -17$$

The left- & right-hand limits disagree \Rightarrow

$$\lim_{x \rightarrow 5} f(x) \nexists$$

$$\lim_{x \rightarrow 5^-} f(x) = -17 \neq 17 = \lim_{x \rightarrow 5^+} f(x) \Rightarrow \lim_{x \rightarrow 5} f(x) \nexists$$

$\lim_{x \rightarrow 5}$ bad

IVT question

$f(x) = \frac{1}{4} \cdot 2^x - x^2 + 4x - 3$ is cont^S on \mathbb{R} ↑

$$\lim(f+g) = \lim f + \lim g$$

b/c $f(x) = \text{poly} + \text{exponential}$ = cont^S + cont^S = cont^S.

$$f(0) = \frac{1}{4} \cdot 2^0 - 0^2 + 4(0) - 3 = \frac{1}{4} - 3 = \frac{1-12}{4} = -\frac{11}{4} < 0$$

$$f(2) = \frac{1}{4} \cdot 2^2 - 2^2 + 4(2) - 3 = 1 - 4 + 8 - 3 = 2 > 0$$

⇒ ∃ c ∈ (0, 2) ∃ f(c) = 0, i.e.

$f(0) = -\frac{11}{4} < 0 < f(2) = 2$ & continuity

⇒ IVT holds.

I cheated you of EVT

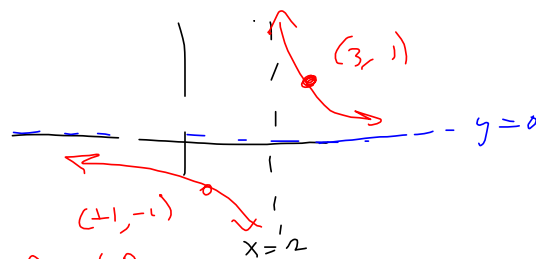
$f(x) = \frac{1}{x-2}$ isn't cont^S

on [0, 4]

$$f(1) = -1$$

$$f(3) = +1$$

but $f(x) \neq 0$
eval. never.



(B2)

$$\sqrt{2x+2h} - \sqrt{2x} = \sqrt{2h}$$

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

$$5 = \sqrt{25} = \sqrt{16+9} = \sqrt{16} + \sqrt{9} = 4+3 = 7$$

$$\overline{a+bi} = a-bi$$

 \sqrt{x} :

$$\left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right) \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) = \frac{x+h - x}{h(\sqrt{x+h} + \sqrt{x})}$$

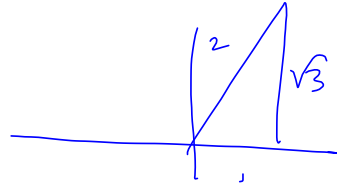
$$(a-b)(a+b) = a^2 - b^2$$

$$\frac{a-b}{\sqrt{a} + \sqrt{b}} = \sqrt{a} - \sqrt{b}$$

$$\frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \frac{1}{\sqrt{x+h} + \sqrt{x}} \xrightarrow{h \rightarrow 0} \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}} = f'(x)$$

Find an equation of the tangent line
to $f(x) = \sin(x)$ @ $x = \frac{\pi}{3}$

$$f\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} = y_1, \quad x_1 = \frac{\pi}{3}$$



$$f'(x) = \cos(x)$$

$$f'\left(\frac{\pi}{3}\right) = \frac{1}{2} = m$$

Linearization of f @ $x = \frac{\pi}{3}$
= Tangent line to $f(x)$ @ $x = \frac{\pi}{3}$

$$\text{is } y = m(x - x_1) + y_1$$

$$= \frac{1}{2}\left(x - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} = L_{\frac{\pi}{3}}(x)$$

Use it to approximate

$$\sin(62^\circ) = \sin\left((62^\circ)\left(\frac{\pi}{180^\circ}\right)\right)$$

$$\begin{aligned} \text{So } L_{\frac{\pi}{3}}(62^\circ) &= \frac{1}{2}\left(\frac{62\pi}{180} - \frac{\pi}{3}\right) + \frac{\sqrt{3}}{2} \\ &= \frac{1}{2}\left(\frac{2\pi}{180}\right) + \frac{\sqrt{3}}{2} = \frac{1}{2}\left[\frac{\pi}{90} + \sqrt{3}\right] \end{aligned}$$

\approx