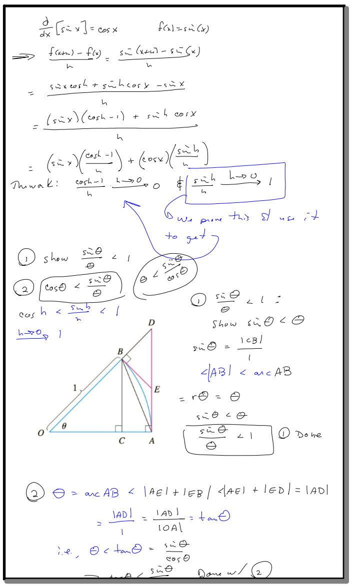


$\frac{d}{dx}[\sin x] = \cos x$ $\quad f(x) = \sin(x)$

$\Rightarrow \frac{f(x+h) - f(x)}{h} = \frac{\sin(x+h) - \sin(x)}{h}$

$= \frac{\sin x \cos h + \sin h \cos x - \sin x}{h}$

$= \frac{(\sin x)(\cos h - 1) + \sin h \cos x}{h}$

$= (\sin x)\left(\frac{\cos h - 1}{h}\right) + (\cos x)\left(\frac{\sin h}{h}\right)$

Thm'k: $\frac{\cos h - 1}{h} \xrightarrow{h \to 0} 0$ & $\boxed{\frac{\sin h}{h} \xrightarrow{h \to 0} 1}$

We prove this & use it to get

① show $\frac{\sin\theta}{\theta} < 1$

② $\boxed{\cos\theta < \frac{\sin\theta}{\theta}}$ $\quad \theta < \frac{\sin\theta}{\cos\theta}$

$\cos h < \frac{\sin h}{h} < 1$

$\xrightarrow{h \to 0} 1$

① $\frac{\sin\theta}{\theta} < 1$:

Show $\sin\theta < \theta$

$\sin\theta = \frac{|CB|}{1}$

$< |AB| < \text{arc } AB$

$= r\theta = \theta$

$\sin\theta < \theta$

$\boxed{\frac{\sin\theta}{\theta} < 1}$ ① Done

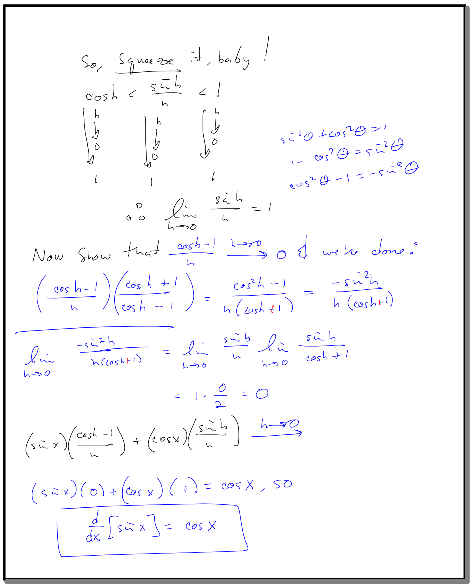
② $\theta = \text{arc } AB < |AE| + |EB| < |AE| + |ED| = |AD|$

$= \frac{|AD|}{1} = \frac{|AD|}{|OA|} = \tan\theta$

i.e., $\theta < \tan\theta = \frac{\sin\theta}{\cos\theta}$

Done w/ ②

Sep 20-8:17 AM



So, Squeeze it, baby!

$\cos h < \frac{\sin h}{h} < 1$

$h \to 0$: $\quad 1 \quad 1 \quad 1$

$\sin^2\theta + \cos^2\theta = 1$

$1 - \cos^2\theta = \sin^2\theta$

$\cos^2\theta - 1 = -\sin^2\theta$

$\therefore \lim_{h \to 0} \frac{\sin h}{h} = 1$

Now show that $\frac{\cos h - 1}{h} \xrightarrow{h \to 0} 0$ & we're done:

$\left(\frac{\cos h - 1}{h}\right)\left(\frac{\cos h + 1}{\cos h - 1}\right) = \frac{\cos^2 h - 1}{h(\cos h + 1)} = \frac{-\sin^2 h}{h(\cos h + 1)}$

$\lim_{h \to 0} \frac{-\sin^2 h}{h(\cos h + 1)} = \lim_{h \to 0} \frac{\sin h}{h} \lim_{h \to 0} \frac{\sin h}{\cos h + 1}$

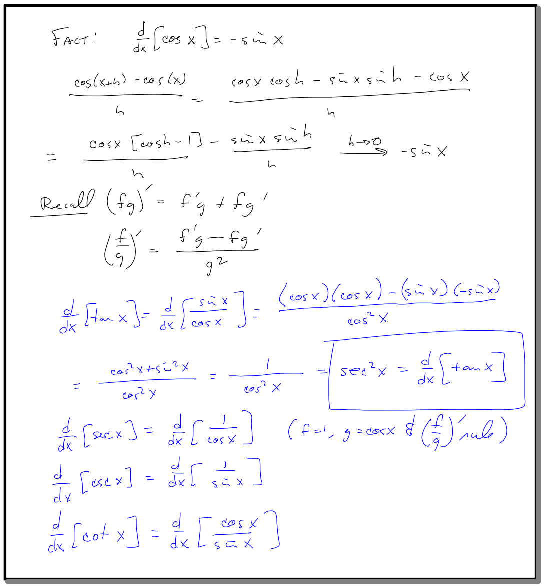
$= 1 \cdot \frac{0}{2} = 0$

$(\sin x)\left(\frac{\cos h - 1}{h}\right) + (\cos x)\left(\frac{\sin h}{h}\right) \xrightarrow{h \to 0}$

$(\sin x)(0) + (\cos x)(1) = \cos x$, so

$\boxed{\frac{d}{dx}[\sin x] = \cos x}$

Sep 20-9:05 AM



FACT: $\frac{d}{dx}[\cos x] = -\sin x$

$\frac{\cos(x+h) - \cos(x)}{h} = \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$

$= \frac{\cos x[\cos h - 1]}{h} - \frac{\sin x \sin h}{h} \xrightarrow{h \to 0} -\sin x$

Recall $(fg)' = f'g + fg'$

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{\cos^2 x}$

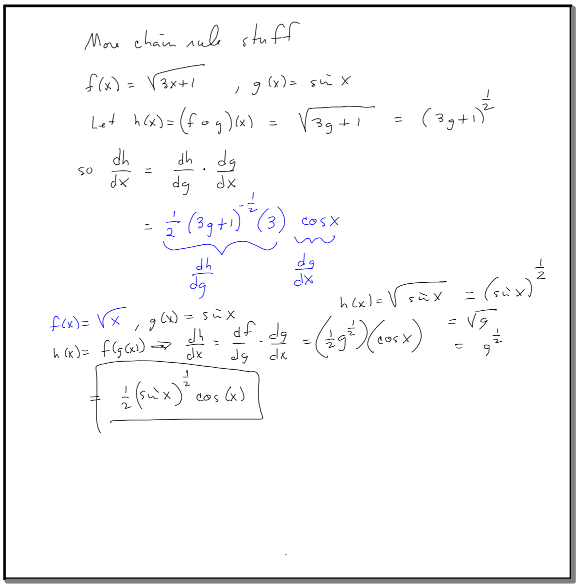
$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x = \frac{d}{dx}[\tan x]}$

$\frac{d}{dx}[\sec x] = \frac{d}{dx}\left[\frac{1}{\cos x}\right]$ $\quad \left(f = 1,\ g = \cos x \ \& \left(\frac{f}{g}\right)' \text{rule}\right)$

$\frac{d}{dx}[\csc x] = \frac{d}{dx}\left[\frac{1}{\sin x}\right]$

$\frac{d}{dx}[\cot x] = \frac{d}{dx}\left[\frac{\cos x}{\sin x}\right]$

Sep 20-8:45 AM



More chain rule stuff

$f(x) = \sqrt{3x+1}$, $\quad g(x) = \sin x$

Let $h(x) = (f \circ g)(x) = \sqrt{3g+1} = (3g+1)^{\frac{1}{2}}$

so $\frac{dh}{dx} = \frac{dh}{dg} \cdot \frac{dg}{dx}$

$= \underbrace{\frac{1}{2}(3g+1)^{-\frac{1}{2}}(3)}_{\frac{dh}{dg}} \underbrace{\cos x}_{\frac{dg}{dx}}$

$h(x) = \sqrt{\sin x} = (\sin x)^{\frac{1}{2}}$

$f(x) = \sqrt{x}$, $\quad g(x) = \sin x$

$h(x) = f(g(x)) \Rightarrow \frac{dh}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx} = \left(\frac{1}{2} g^{\frac{1}{2}}\right)(\cos x)$ $\quad = \sqrt{g} = g^{\frac{1}{2}}$

$= \boxed{\frac{1}{2}(\sin x)^{\frac{1}{2}} \cos(x)}$

Sep 20-8:53 AM