$\frac{d}{dx}[x^n]$ = derivative $\underbrace{\text{with respect to}}_{\text{wrt}}$ x of x^n

(Liebniz notation)

$= nx^{n-1}$ $\quad (\forall n \in \mathbb{N} = \{1,2,3,\dots\}$ (for now))

Will see this is the case $\forall n \neq 0$)

Recall: $x^2 - a^2 = (x-a)(x+a)$

$x^n - a^n = (x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-1})$

$x^n - 1 = (x-1)(x^{n-1} + x^{n-2} + \dots + 1)$

$x^3 - 1 = (x-1)(x^2 + x + 1)$

$x^3 - 8 = (x-2)(x^2 + 8x + 8^2)$

$f(x) = x^n \Rightarrow \frac{f(x)-f(a)}{x-a} \xrightarrow{x \to a} f'(a)$

$$\frac{f(x)-f(a)}{x-a} = \frac{x^n - a^n}{x-a} = \frac{(x-a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \dots + a^{n-1})}{x-a}$$

$$= x^{n-1} + ax^{n-2} + \dots + a^{n-1} \xrightarrow{x \to a} a^{n-1} + a \cdot a^{n-2} + a^2 \cdot a^{n-3} + \dots a^{n-1}$$

$(x \neq a)$

$$= \underbrace{a^{n-1} + a^{n-1} + \dots + a^{n-1}}_{n \text{ of 'em}} = na^{n-1}$$

which proves the formula (

$\frac{d}{dx}[x^5] = 5x^4$ = slope of x^5 @ x !

Props of limits

$3-2 = 3+(-2)$

$\lim (f+g) = \lim f + \lim g$

$\lim (cf) = c \lim f$

$\lim (fg) = (\lim f)(\lim g)$

$\lim \left(\frac{f}{g}\right) = \frac{\lim f}{\lim g}$

Props of derivatives.

$\frac{d}{dx}[f+g] = \frac{df}{dx} + \frac{dg}{dx} = f' + g'$

$(f+g)' = f' + g'$

$(cf)' = cf'$

$(fg)' = f'g + fg'$ us

$f'g + g'f$ Book

$gf' + fg'$

Product Rule

$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ Quotient Rule.

$$\lim_{x \to a} (f(g(x))) = f(\lim_{x \to a} g(x))$$

$$\boxed{(f(g))' = \frac{d}{dx}[f(g(x))] = \frac{df}{dg} \cdot \frac{dg}{dx}}$$

Chain Rule

How fast f changes wrt g

How fast g changes wrt x

$$\lim_{x \to 2} (3x) = 3\lim_{x \to 2} x = 3(2)$$

Book's going to keep you away from Product, Quotient and Chain Rules 'til later.

x^2+1

$f(x) = x^2+1$, $g(x) = 3x$

$f \circ g = f(g(x)) = (3x)^2+1 = 9x^2+1 = g^2+1$

(A) $\frac{d}{dx}[f(g(x))] = \frac{df}{dg} \cdot \frac{dg}{dx} = (2g)(3) = 6g = 6(3x) = 18x$

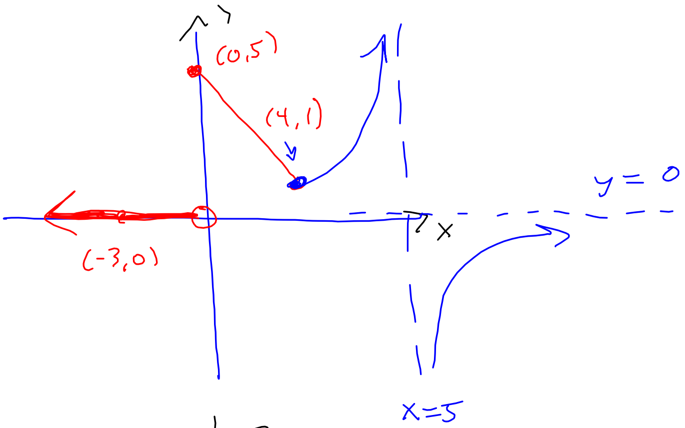
(B) $\frac{d}{dx}[9x^2+1] = 18x^1$

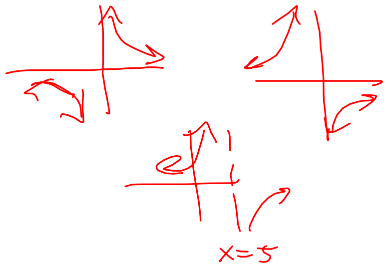
$1 \cdot 3x^0 = 3$

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 5-x & \text{if } 0 \le x < 4 \\ \frac{1}{5-x} & \text{if } 4 \le x \end{cases}$$

$$\frac{1}{5-x} = -\frac{1}{x-5}$$

f's pieces are cnt-s





$$\frac{-1}{4-5} = \frac{-1}{-1} = 1$$

$$f_-'(4) = \frac{d}{dx}\left[5-x\right] = -1$$

$$f_+'(4) = \frac{d}{dx}\left[\frac{1}{5-x}\right]$$

$g' = 0$, $h' = -1$

Cheat & use quotient rule: $g = 1$, $h = 5-x$

$$\left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2} = \frac{0\cdot(5-x) - 1\cdot(-1)}{(5-x)^2} = \frac{1}{(x-5)^2}$$

$(x-5)^2$

& so

$$\boxed{\begin{aligned} f_+'(4) &= \frac{1}{(4-5)^2} = +1 \\ f_-'(f) &= -1 \end{aligned}} \quad \boxed{f'(4) \nexists}$$

$$(5-x)^2 = \left((-1)(x-5)\right)^2$$
$$= (-1)^2(x-5)^2 = (x-5)^2$$