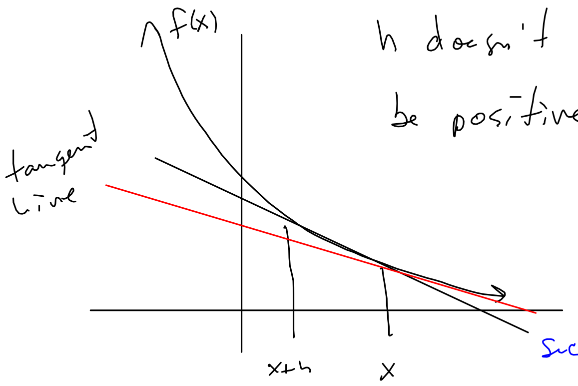


h doesn't have to be positive.

$$m_{sec} = \frac{f(x+h)-f(x)}{h}$$

$$h \to 0 \Rightarrow m_{tan}$$

secant line

LCD $= (x-3)(x+h-3)$

$f(x) = \dfrac{x+2}{x-3}$ Find $f'(x)$

$$\frac{f(x+h)-f(x)}{h} = \frac{\dfrac{(x+h)+2}{(x+h)-3} - \dfrac{x+2}{x-3}}{h} = \frac{1}{h}\left[\frac{(x+h+2)(x+h-3) - (x+2)(x-3)}{(x-3)(x+h-3)}\right]$$

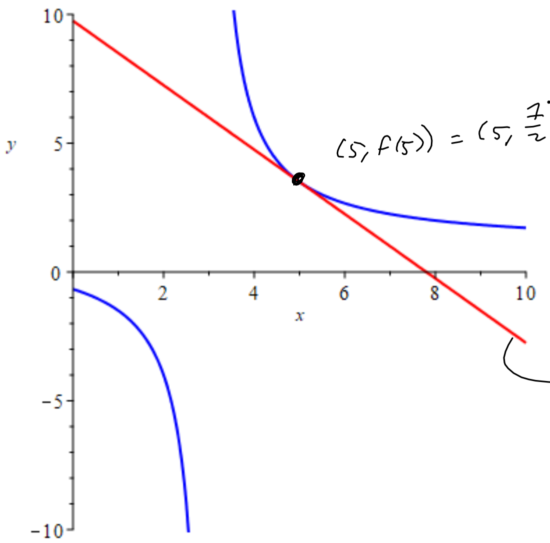
scratch:

$$(x+h+2)(x+h-3)$$
$$= x^2 + xh - 3x + hx + h^2 - 3h + 2x + 2h - 6$$
$$= x^2 + 2xh - x + h^2 - h - 6$$

$$(x+2)(x-3) = x^2 - x - 6$$

$$= \frac{1}{h}\left[\frac{x^2 + 2xh - x + h^2 - h - 6 - x^2 + x + 6}{(x-3)(x+h-3)}\right] = \left[\frac{2xh + h^2 - h}{(x-3)(x+h-3)}\right]\left(\frac{1}{h}\right)$$

$$= \frac{2x+h-1}{(x-3)(x+h-3)} \xrightarrow{h \to 0} \boxed{\frac{2x-1}{(x-3)^2} = f'(x)}$$



$(5, f(5)) = (5, \frac{7}{2})$

$$f'(5)(x-5) + f(5)$$
$$= \boxed{-\frac{5}{4}(x-5) + \frac{7}{2} = L(x)}$$

STOP RIGHT HERE

$$= -\frac{5}{4}x + \frac{25}{4} + \frac{14}{4}$$

$$\boxed{L(x) = -\frac{5}{4}x + \frac{39}{4}}$$ optional.

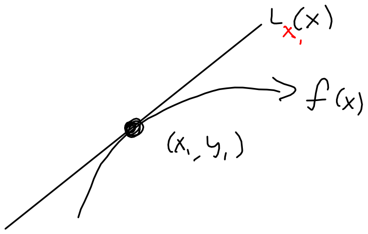
Some of the graphs for homework are a pain. Manage your time wisely. Don't just do busy work to please me, if you have the concept nailed.

Line with slope m thru (x_1, y_1) is

$$y = m(x - x_1) + y_1$$

Tangent line to $f(x)$ @ (x_1, y_1) is

$$y = L(x) = f'(x_1)(x - x_1) + f(x_1)$$



Smooth functions are "locally linear."

Prove that $\lim_{x \to 2} (x^2 - 5) = -1$

Scratch:

$|x^2 - 5 - (-1)| = |x^2 - 4| = \underbrace{|x-2|}_{<\delta} \underbrace{|x+2|}_{\text{ceiling on this.}}$

$$\boxed{\begin{array}{l} \text{Assume } \delta \le 1: \quad \text{if } x \to 2 \\ 1 < x < 3 \\ 1+2 < x+2 < 3+2 \\ 3 < x+2 < 5 \\ \text{so } |x+2| < 5 \end{array}}$$

So $< |x-2| \cdot 5 < 5\delta \le \varepsilon \longrightarrow \delta \le \frac{\varepsilon}{5}$

Proof: Let $\varepsilon > 0$ be given. Define $\boxed{\delta = \min\left\{1, \frac{\varepsilon}{5}\right\}}$

Then $0 < |x-2| < \delta$ implies

$|x^2 - 5 - (-1)| = |x^2 - 4| = |x+2||x-2| < |x+2|\delta$

$< 5\delta \le 5 \cdot \frac{\varepsilon}{5} = \varepsilon$ ∎