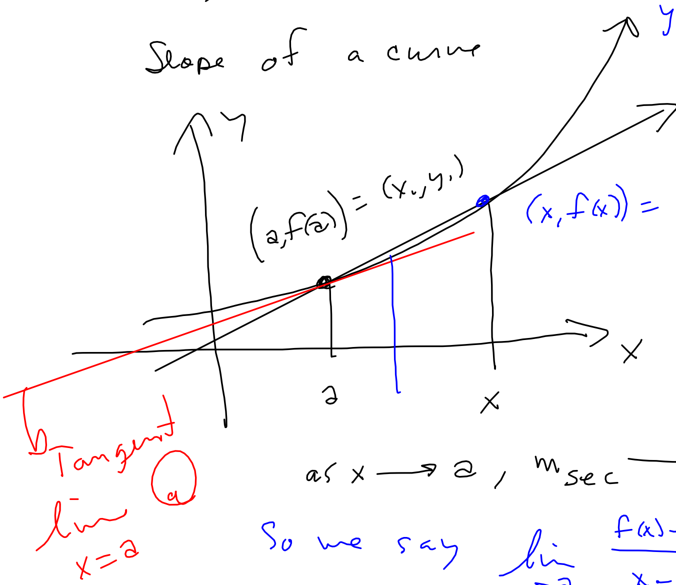
§2.1, 2.2

Jonathan, Owen, Pacen   Test1's

Slope of a curve



Secant Line has slope $m = \frac{y_2-y_1}{x_2-x_1} = \frac{f(x)-f(a)}{x-a}$

It approximates the slope of f(x) @ x = a.

as $x \to a$, $m_{sec} \to m_{tan}$

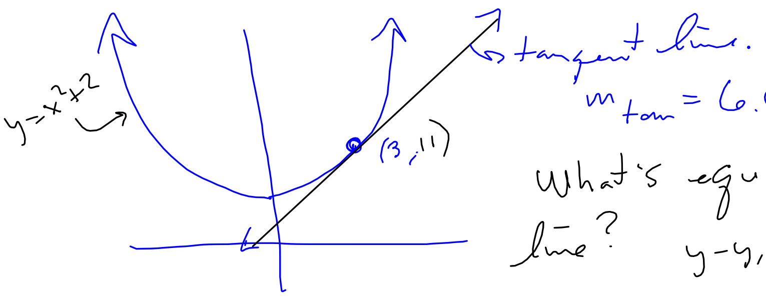
So we say $\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = m_{tan} = f'(a)$,

where f' = the derivative = the derived function that gives the slope of f @ x.

$f(x) = x^2+2$. Find $f'(3)$:

$$\lim_{x\to 3} \frac{f(x)-f(3)}{x-3} = \lim_{x\to 3} \frac{x^2+2-(3^2+2)}{x-3} = \lim_{x\to 3} \frac{x^2+2-9-2}{x-3}$$

$$= \lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3} (x+3) = 6.$$



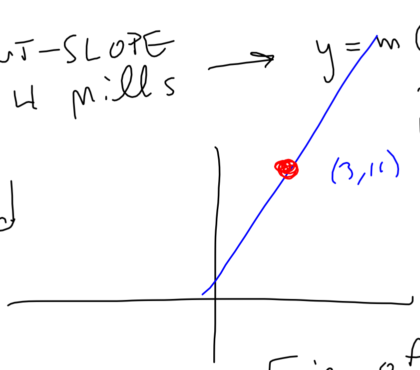
$m_{tan} = 6$ @ $x=3$

What's equation of that line? $y - y_1 = m(x - x_1)$

POINT-SLOPE
4 Mills $\rightarrow$ $y = m(x - x_1) + y_1$

$$\boxed{y = 6(x-3)+11}$$

Tangent Line for $f(x)$ thru $(3,11) = (3, f(3))$

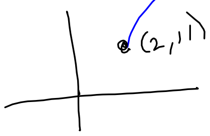


$g(x) = a f(bx + c) + d$

$= a f\left(b\left(x + \frac{c}{b}\right)\right) + d$

$3\sqrt{5(x-2)} + 11$

$\sqrt{x-2} + 11$



Eq'n of tangent to $f(x)$ @ $x = 3$, $y = 11$ is:

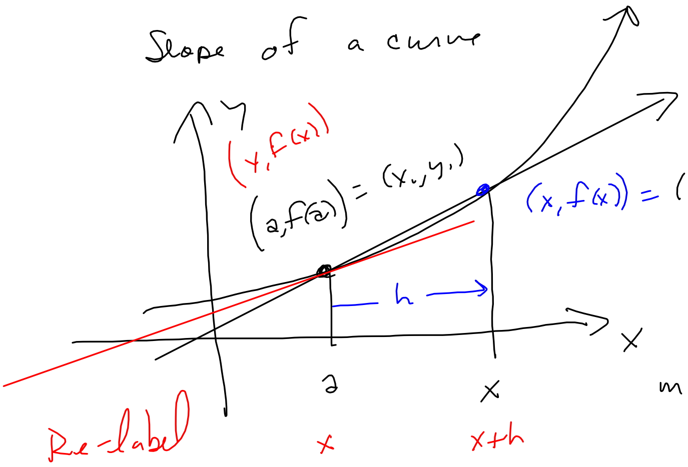
$$y = f'(3)(x-3) + f(3)$$
$$= m(x-a) + f(a)$$

$$\boxed{L_a(x) = f'(a)(x-a) + f(a)}$$

linearization of $f(x)$ @ $x = a$

We have $f'(a)$.
Let's talk about $f'(x)$, in general.

Slope of a curve



Secant Line

$x_2 - x_1 =$

$*\ x+h-x = h$

$(x, f(x)) = (x_2, y_2)$ "=" $(x+h, f(x+h))$

$$m_{sec} = \frac{y_2-y_1}{x_2-x_1} = \frac{f(x+h)-f(x)}{h^*}$$

$$m_{tan} = \lim_{h\to 0} \frac{(x+h)^2+2-(x^2+2)}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+2-x^2-2}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h}$$

$$= \lim_{h\to 0} (2x+h) = \boxed{2x = f'(x)}$$

$= m_{tan}$

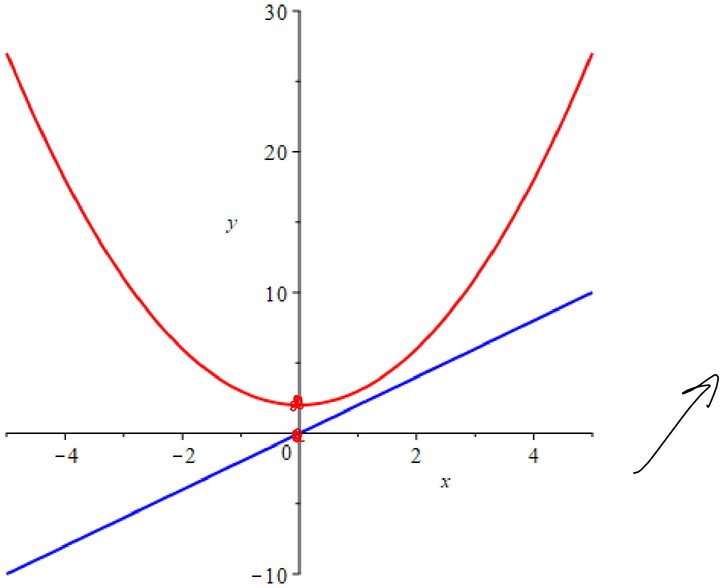
$f(x) = x^2+2$

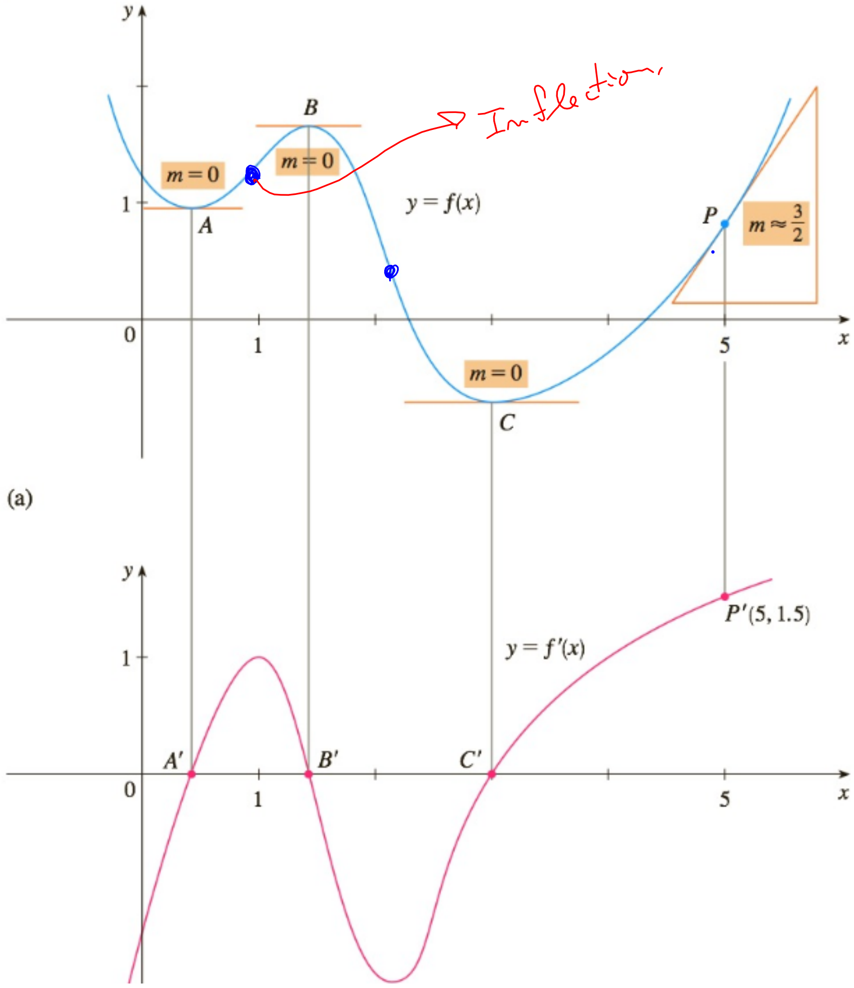
$f(☺) = ☺^2+2$

$f(\square) = \square^2+2$

$f(x+h) = (x+h)^2+2$

Tangent line: $y = f'(3)(x-3) + f(3)$


30
20
10
y
−10
−4
−2
0
2
4
x


y
Inflection
B
m = 0
m = 0
1
A
y = f(x)
P
m ≈ 3/2
0
1
5
x
m = 0
C
(a)
y
P′(5, 1.5)
y = f′(x)
1
A′
B′
C′
0
1
5
x