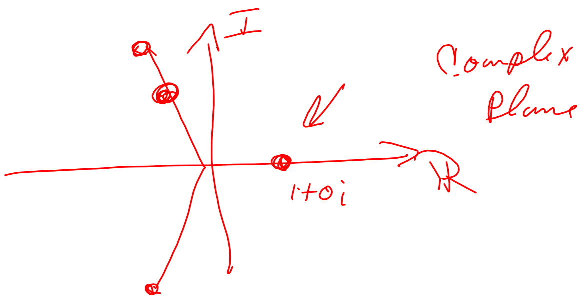
$$\lim_{t \to 1} \frac{t^4-1}{t^3-1} = ?$$

~~$f(x)$~~ $f(t) = \dfrac{t^4-1}{t^4-1}$

$$\frac{t^4-1}{t^3-1} = \frac{(t^2-1)(t^2+1)}{(t-1)(t^2+t+1)} = \frac{\cancel{(t-1)}(t+1)(t^2+1)}{\cancel{(t-1)}(t^2+t+1)} \xrightarrow{t \to 1} \frac{(2)(1^2+1)}{(1^2+1+1)}$$

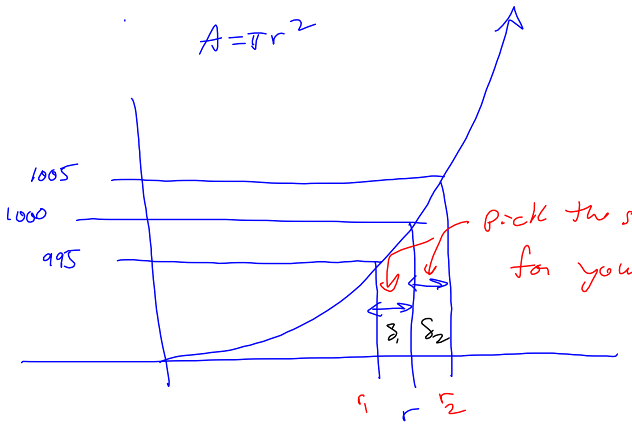
$$= \boxed{\frac{4}{3} = \lim_{t \to 1} f(t)}$$



§1.7 # 11 $A = 1000 \pm 5\ cm^2$

Find δ for $\varepsilon = 5$ cm

$A = \pi r^2$



Pick the smaller for your δ

You find r_1 & r_2 & r!

$\pi r^2 = 1000$

$r^2 = \frac{1000}{\pi}$

$r = \pm\sqrt{\frac{1000}{\pi}}$ Take $\boxed{r = +\sqrt{\frac{1000}{\pi}}}$ $\boxed{r \approx 17.84124116}$

$\pi r_2^2 = 1005$

$r_2^2 = \frac{1005}{\pi}$

$r_2 = \pm\sqrt{\frac{1005}{\pi}} \approx \pm 17.88578865 \rightarrow \boxed{r_2 \approx 17.88578865}$

$\vdots$

$r_1 \approx \sqrt{\frac{995}{\pi}} \approx \boxed{17.79658216 \approx r_1}$

Now find min of

$\delta_1 = \sqrt{\frac{1000}{\pi}} - \sqrt{\frac{995}{\pi}} \approx .04465900$

$\delta_2 = \sqrt{\frac{1005}{\pi}} - \sqrt{\frac{1000}{\pi}} \approx .04454749$

$\delta_2 < \delta_1$

Let $\boxed{\delta = .0445}$

Round Down!

(Want to be conservative on δ)

Fall '17 Test 1

1 (a) $f(x) = 2x^2+3x$ Find m_{sec} from $(2, f(2))$ to $(2.001, f(2.001))$

$$m = \frac{f(2.001)-f(2)}{2.001-2} = \frac{y_2-y_1}{x_2-x_1} = \frac{2(2.001)^2+3(2.001)-(3(2)^2+3(2))}{.001}$$

$f(2.001)$ is a pain. I bet $m \approx 11.002$

(b) $f'(2)$ = actual slope @ $x=2$:

$$\lim_{h\to 0} \frac{f(2+h)-f(2)}{h}$$ is the actual.

$$\frac{2(x+h)^2+3(x+h)-[2x^2+3x]}{h}$$

$$= \frac{2(x^2+2xh+h^2)+3x+3h-2x^2-3x}{h}$$

$$= \frac{2x^2+4xh+2h^2+3h-2x^2}{h} = \frac{4xh+2h^2+3h}{h}$$

$$= \frac{h(4x+2h+3)}{h} = 4x+2h+3 \quad (h \neq 0) \xrightarrow{h\to 0} 4x+3 = f'(x)$$

$\Rightarrow f'(2) = 8+3 = 11$ is what we want

#9

$$\sin\left(\frac{\pi}{3}x\right) = x-1 \Rightarrow$$

$$\sin\left(\frac{\pi}{3}x\right) - x + 1 = 0$$

Let $f(x) = \sin\left(\frac{\pi}{3}x\right) - x + 1$

$\frac{\pi}{3x}$ $(\pi/3)x$

Prove $\exists\ c \ni f(c) = 0$ $(c \in (0,3))$

$f(0) = 0 - 0 + 1 = 1 > 0$

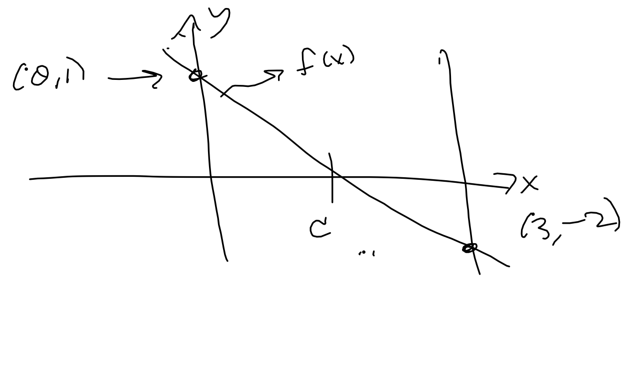
$f(3) = \sin\left(\frac{\pi}{3}(3)\right) - 3 + 1$

$= 0 - 3 + 1 = -2 < 0$

$f(0) = 1 > 0 > -2 = f(3)$

So, by Intermediate Value Theorem (IVT)

$\exists\ c \in (0,3) \ni f(c) = 0$



Claim: $\lim_{x \to 2} (3x-7) = -1$

$3 = m =$ growth rate. Let $\delta = \frac{\varepsilon}{3}$

Proof Let $\varepsilon > 0$ be given. Define $\delta = \frac{\varepsilon}{3}$

Then $0 < |x-2| < \delta \implies$

$$|3x-7-(-1)| = |3x-7+1| = |3x-6| = 3|x-2|$$

$$< 3\delta = 3 \cdot \frac{\varepsilon}{3} = \varepsilon \quad \blacksquare$$