$$\lim_{x\to 1} \frac{x^2-3x+2}{|x-1|}$$

$$f(x) = \frac{x^2-3x+2}{|x-1|} = \begin{cases} \dfrac{x^2-3x+2}{x-1} & \text{if } x-1 \geq 0 \\ \dfrac{x^2-3x+2}{-(x-1)} & \text{if } x-1 < 0 \end{cases}$$

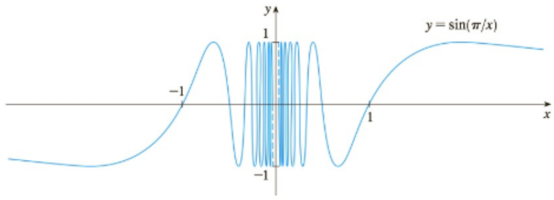
$$= \begin{cases} \dfrac{x^2-3x+2}{x-1} & \text{if } x \geq 1 \\ -\dfrac{x^2-3x+2}{x-1} & \text{if } x < 1 \end{cases}$$

$$x < 1 \Rightarrow f(x) = -\frac{x^2-3x+2}{x-1} = -\frac{(x-2)\cancel{(x-1)}}{\cancel{x-1}} = -(x-2)$$

$$\xrightarrow{x \to 1^-} -(1-2) = +1$$

$$x \geq 1 \Rightarrow f(x) = \frac{x^2-3x+2}{x-1} = \dots = +(x-2) \xrightarrow{x \to 1^+} -1$$

$$-1 \neq +1 \Rightarrow \boxed{\lim f \ \nexists}$$

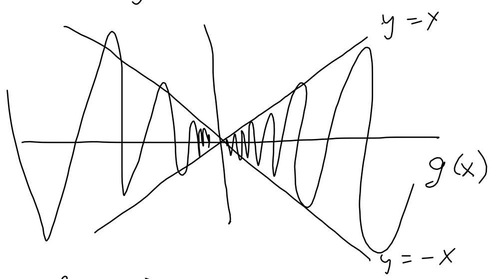


$f(x) = \sin\left(\frac{\pi}{x}\right)$
cnt^s everywhere
except @ $x = 0$.

we can Make it 1-to-1 @ $x=0$, this way

$g(x) = x \sin\left(\frac{\pi}{x}\right)$

Because $-1 \le \sin(*) \le 1$,
the "damping function" $y = x$, pushes the whole thing to zero as $x \to 0$



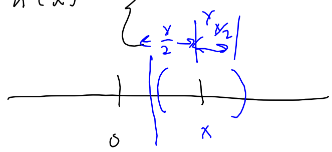
Showed
$\lim_{x\to 0} g(x) = 0$
when we talked about squeeze theorem.

So, $\lim_{x\to 0} g(x) = 0$

even though $g(0) \nexists$

We MAKE a continuous function thus:

$$h(x) = \begin{cases} x \sin\left(\frac{\pi}{x}\right) & \text{if } x \neq 0 \\ \end{cases}$$



f, g cnt^s $\Rightarrow$ fg cnt^s

$f(g(x))$ & $f \circ g$ cnt^s as long as f is cnt^s @ $g(x)$.

(f can eat the range of g)

$\left|\sin\left(\frac{\pi}{x}\right)\right| \le 1$

$\left|x \sin\left(\frac{\pi}{x}\right)\right| \le |x|$

$-x \le x \sin\left(\frac{\pi}{x}\right) \le x$

$x \to 0$: $0 \quad 0 \quad 0$

§1.7

One limit on an $ax+b$ (linear)

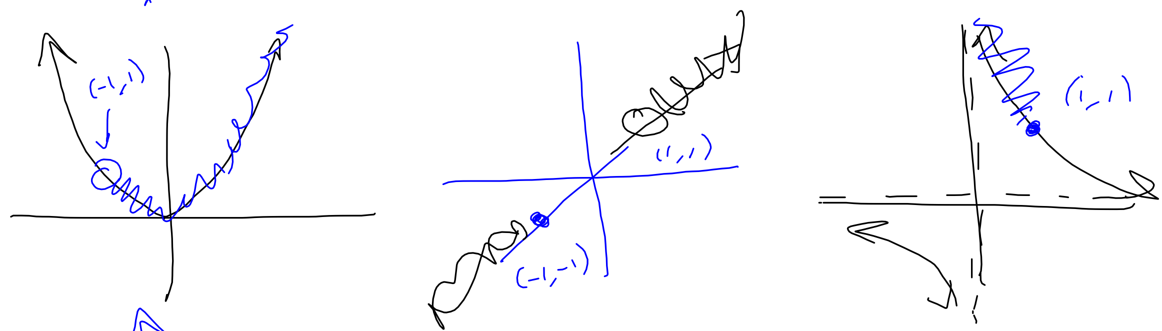
Define $\delta = \frac{\varepsilon}{a}$

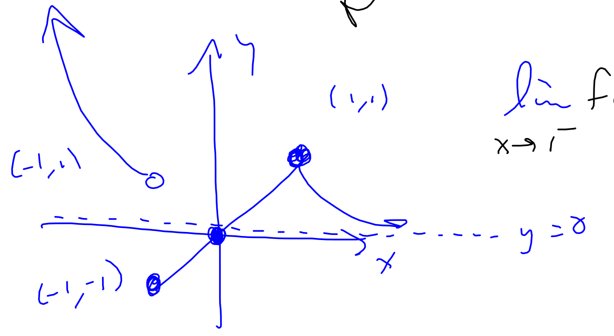
41–43 Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left, or neither? Sketch the graph of f.

41. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ x & \text{if } -1 \leqslant x < 1 \\ 1/x & \text{if } x \geqslant 1 \end{cases}$

$x < -1$ | $-1 \le x < 1$ | $x \ge 1$

x^2 | x | $\frac{1}{x}$





$\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} x = 1$

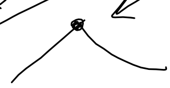
$\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \frac{1}{x} = 1$

$f(1) = 1$

So continuous.

cnts on $(-\infty,-1) \cup (-1, \infty)$

cnts from left:
$\lim\limits_{x\to -1^-} f(x) = +1$ so, nowhere
But $f(-1) = -1$



Called a "cusp"
Can't determine the slope, here.

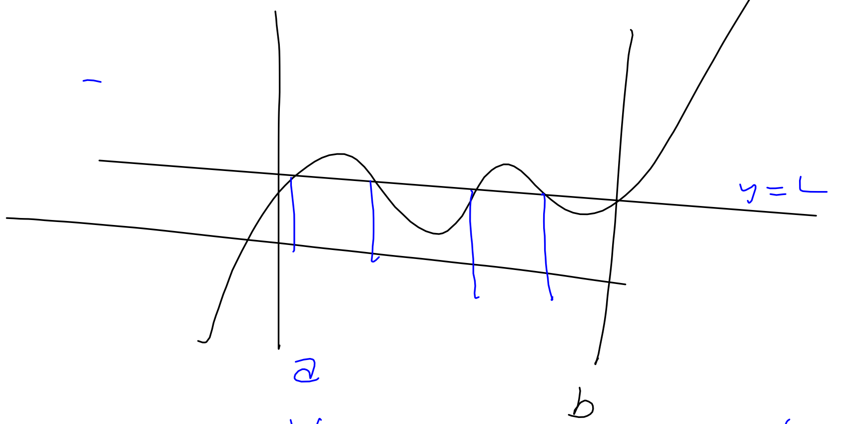
But $\lim\limits_{x\to -1^+} f(x) = -1 = f(-1)$, so, cnts from right at $x=-1$.

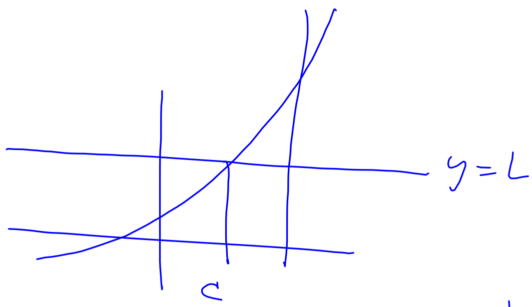
I.V.T f cnt^s on $[a,b]$

& $f(a) < L < f(b)$

(or $f(a) > L > f(b)$)

$\exists\ c \in (a,b) \ni f(c) = L$





IVT & Extreme value Theorem.

$\boxed{\text{EVT \& IVT}}$