Limits at Infinity (§3.4, I think) In context of rationalizing denominators

https://harryzaims.com/201/videos/chapter-03/3-4/3-4-notes.pdf

Brain short-circuit.

$(a-b)(a+b) = a^2 - b^2$

$$\lim_{x\to\infty}\left(\sqrt{9x^2+x} - 3x\right)$$

$$= \frac{\left(\sqrt{9x^2+x} - 3x\right)\left(\sqrt{9x^2+x} + 3x\right)}{\sqrt{9x^2+x} + 3x} = \frac{9x^2 + x - 9x^2}{\sqrt{9x^2+x} + 3x} = \frac{x}{\sqrt{9x^2\left(1 + \frac{1}{9x}\right)} + 3x}$$

$x \to \infty$ ⇒ $x > 0$, so $|x| = x$

$$= \frac{x}{|3x|\sqrt{1+\frac{1}{9x}} + 3x} = \frac{x}{3x\sqrt{1+\frac{1}{9x}} + 3x} = \frac{x}{3x\left(\sqrt{1+\frac{1}{9x}} + 1\right)}$$

$$= \frac{1}{3\left(\sqrt{1+\frac{1}{9x}} + 1\right)} \xrightarrow{x\to\infty} \frac{1}{3(\sqrt{1} + 1)} = \frac{1}{3(2)} = \frac{1}{6}$$

($\frac{1}{9x} \to 0$)

$$\sqrt{9x^2+x} - 3x = \sqrt{9x^2\left(1+\frac{1}{9x}\right)} - 3x$$

$$= |3x|\sqrt{1+\frac{1}{9x}} - 3x = \left(3x\sqrt{1+\frac{1}{9x}} - 3x\right)$$

$$= 3x\left(\sqrt{1+\frac{1}{9x}} + 1\right) =$$